Boolean Algebra

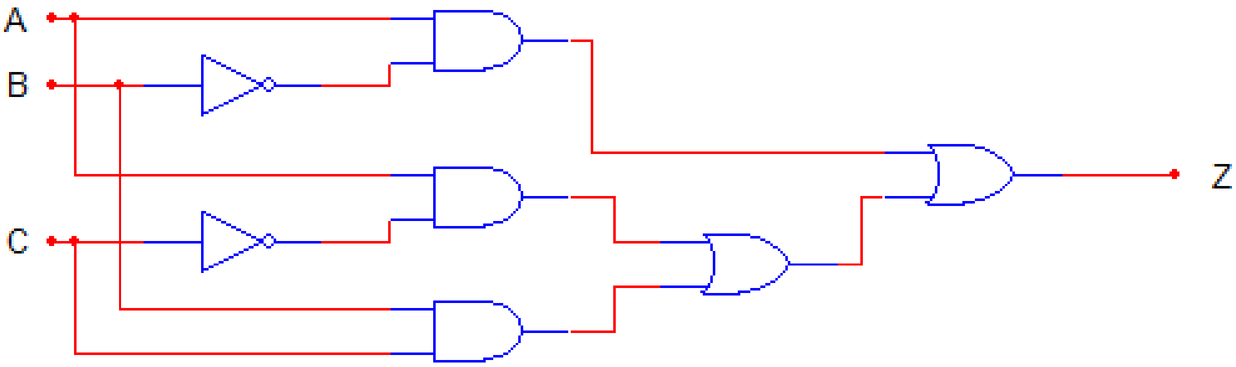
Digital Electronics

What is Boolean Algebra?

Boolean Algebra is a mathematical technique that provides the ability to algebraically **simplify logic expressions**. These simplified expressions will result in a logic circuit that is **equivalent** to the original circuit, yet requires **fewer** gates.

What is Boolean Algebra ?

Before **simplification**



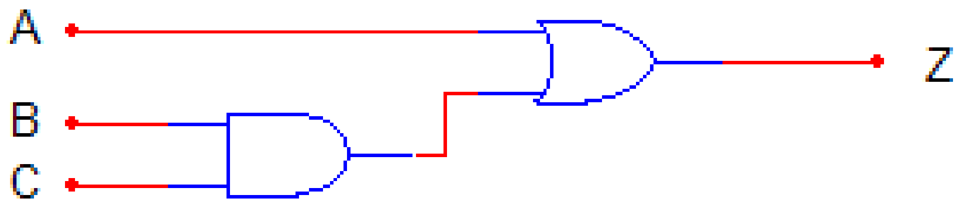
$$Z = BC + A\overline{B} + A\overline{C}$$

After **simplification**

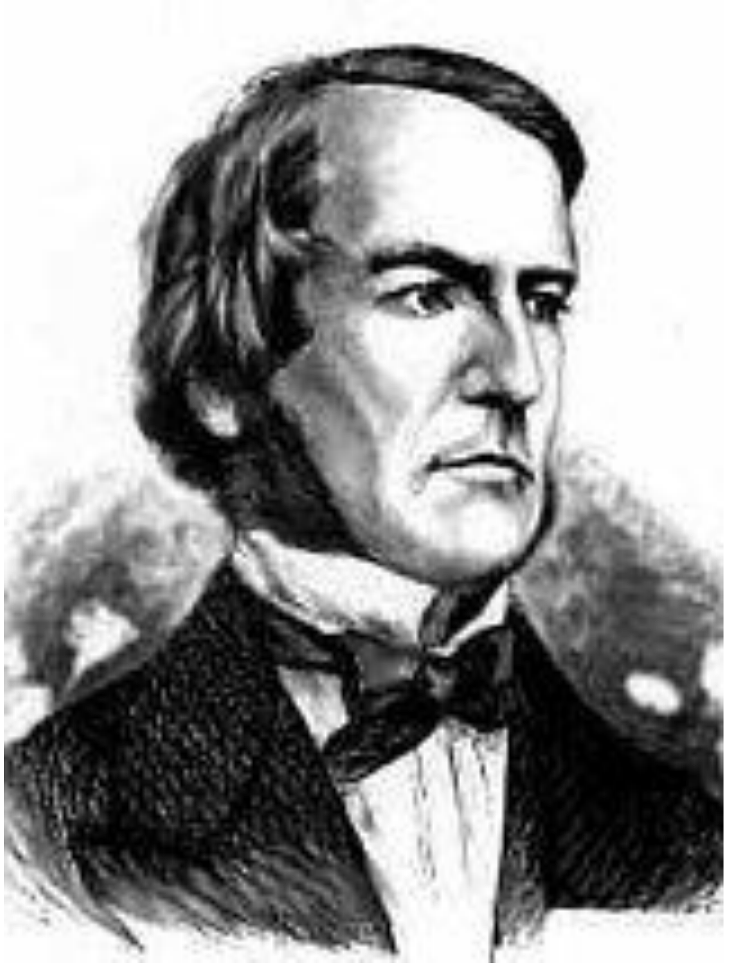
$$Z = BC + A\overline{B} + A\overline{C}$$

Simplification With Boolean Algebra

$$Z = A + BC$$



George Boole



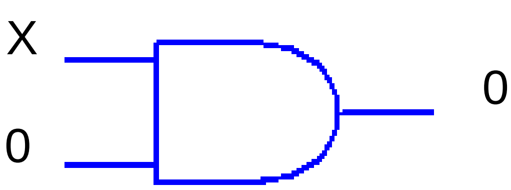
George Boole lived in England in the 19th century. His work on mathematical **logic**, **algebra**, and the **binary** number system has had a unique influence upon the development of **computers**. Boolean Algebra is named after him.

Boolean Theorems (1 of 9)

Single Variable - **AND** Function

Theorem #1

$$\underline{X \cdot 0 = 0}$$



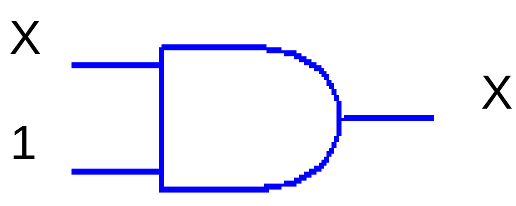
X	**Y**	**Z**
0	0	0
~~0~~	~~1~~	~~0~~
1	0	0
~~1~~	~~1~~	~~1~~

Boolean Theorems (2 of 9)

Single Variable - **AND** Function

Theorem #2

$$X \cdot 1 = X$$



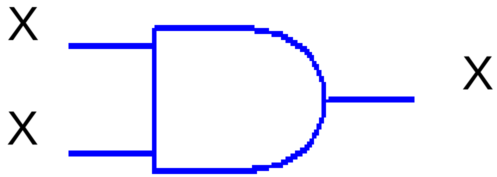
X	**Y**	**Z**
~~0~~	~~0~~	~~0~~
0	1	0
~~1~~	~~0~~	~~0~~
1	1	1

Boolean Theorems (3 of 9)

Single Variable - **AND** Function

Theorem #3

$$X \cdot X = X$$



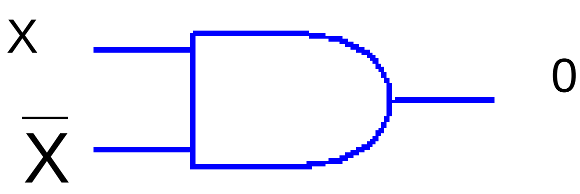
X	Y	Z
0	0	0
~~0~~	~~1~~	~~0~~
~~1~~	~~0~~	~~0~~
1	1	1

Boolean Theorems (4 of 9)

Single Variable - **AND** Function

Theorem #4

$$X \cdot \overline{X} = 0$$



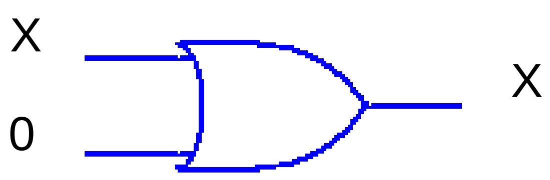
X	Y	Z
~~0~~	~~0~~	~~0~~
0	1	0
1	0	0
~~1~~	~~1~~	~~1~~

Boolean Theorems (5 of 9)

Single Variable - **OR** Function

Theorem #5

$$X + 0 = X$$



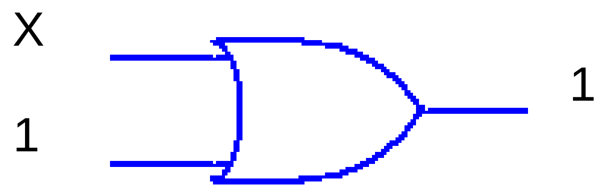
X	Y	Z
0	0	0
~~0~~	~~1~~	~~1~~
1	0	1
~~1~~	~~1~~	~~1~~

Boolean Theorems (6 of 9)

Single Variable - **OR** Function

Theorem #6

$$X + 1 = 1$$



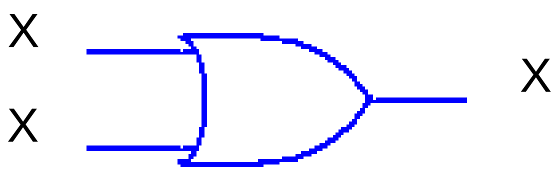
X	Y	Z
~~0~~	~~0~~	~~0~~
0	1	1
~~1~~	~~0~~	~~1~~
1	1	1

Boolean Theorems (7 of 9)

Single Variable - **OR** Function

Theorem #7

$$X + X = X$$



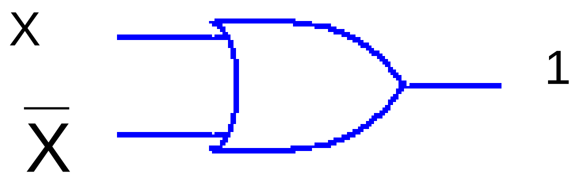
X	Y	Z
0	0	0
~~0~~	~~1~~	~~1~~
~~1~~	~~0~~	~~1~~
1	1	1

Boolean Theorems (8 of 9)

Single Variable - **OR** Function

Theorem #8

$$X + \overline{X} = 1$$



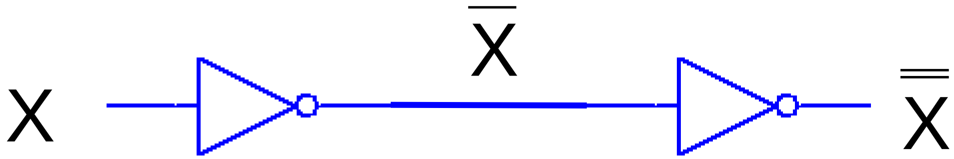
X	Y	Z
~~0~~	~~0~~	~~0~~
0	1	1
1	0	1
~~1~~	~~1~~	~~1~~

Boolean Theorems (9 of 9)

Single Variable - **Invert** Function

Theorem #9

$$\overline{\overline{X}} = X$$



X	$\overline{X}$	$\overline{\overline{X}}$
0	1	0
1	0	1

Example #1: Boolean Algebra

Simplify the following Boolean expression and note the Boolean theorem used at each step.
Put the answer in SOP form.

$$F_1 = \mathrm{A\,A\,B} + \mathrm{C\,\overline{C}\,D}$$

Step #1: Boolean Algebra

$$F_1 = A\,A\,B + C\,\overline{C}\,D$$

Which Theorem can be applied to AAB?

Step #1: Boolean Algebra

$$F_1 = A\,A\,B + C\,\overline{C}\,D$$

Which Theorem can be applied to AAB?

Theorem # **3**:

Step #1: Boolean Algebra

$$F_1 = A\,A\,B + C\,\overline{C}\,D$$

Which Theorem can be applied to AAB?

Theorem # **3:** $\underline{X \cdot X = X}$

Step #1: Boolean Algebra

$$F_1 = A\,A\,B + C\,\overline{C}\,D$$

Which Theorem can be applied to AAB?

Theorem # **3:** $X \cdot X = X$

AAB can be simplified to **AB**

Step #1: Boolean Algebra

$$F_1 = A\,A\,B + C\,\overline{C}\,D$$

Which Theorem can be applied to AAB?

Theorem # **3:** $X \cdot X = X$

AAB can be simplified to **AB**

$$\underline{F_1 = A\,B + C\,\overline{C}\,D}$$

Step #2: Boolean Algebra

$$F_1 = \mathrm{A\,B} + \mathrm{C\,\overline{C}\,D}$$

Step #2: Boolean Algebra

$$F_1 = AB + C\overline{C}D$$

Which Theorem can be applied to $C\overline{C}D$?

Step #2: Boolean Algebra

$$F_1 = A\,B + C\,\overline{C}\,D$$

Which Theorem can be applied to $C\,\overline{C}\,D$?

Theorem # **4:**

Step #2: Boolean Algebra

$$F_1 = AB + C\overline{C}D$$

Which Theorem can be applied to $C\overline{C}D$?

Theorem # **4:** $\underline{X \cdot \overline{X} = 0}$

Step #2: Boolean Algebra

$$F_1 = AB + C\overline{C}D$$

Which Theorem can be applied to $C\overline{C}D$?

Theorem # **4:** $X \cdot \overline{X} = 0$

$\underline{C\overline{C}D}$ can be simplified to **0D**

Step #2: Boolean Algebra

$$F_1 = AB + C\overline{C}D$$

Which Theorem can be applied to $C\overline{C}D$?

Theorem # **4:** $X \cdot \overline{X} = 0$

$C\overline{C}D$ can be simplified to **0D**

$$\underline{F_1 = AB + 0D}$$

Step #3: Boolean Algebra

$$F_1 = AB + 0D$$

Step #3: Boolean Algebra

$$F_1 = AB + 0D$$

Which Theorem can be applied to **<u>0D</u>** ?

Step #3: Boolean Algebra

$$F_1 = A\,B + 0\,D$$

Which Theorem can be applied to **0D** ?

Theorem # **1**:

Step #3: Boolean Algebra

$$F_1 = AB + 0D$$

Which Theorem can be applied to **0D** ?

Theorem # **1**: $\underline{X \cdot 0 = 0}$

Step #3: Boolean Algebra

$$F_1 = AB + 0D$$

Which Theorem can be applied to **0D** ?

Theorem # **1**: $X \cdot 0 = 0$

0D can be simplified to **0**

Step #3: Boolean Algebra

$$F_1 = A\,B + 0\,D$$

Which Theorem can be applied to **0D** ?

Theorem # **1**: $$X \cdot 0 = 0$$

0D can be simplified to **0**

$$\underline{F_1 = A\,B + 0}$$

Step #4: Boolean Algebra

$$F_1 = AB + 0$$

Which Theorem can be applied to **AB + 0** ?

Step #4: Boolean Algebra

$$F_1 = \text{A B} + 0$$

Which Theorem can be applied to **AB + 0** ?

Theorem # **5**:

Step #4: Boolean Algebra

$$F_1 = AB + 0$$

Which Theorem can be applied to **AB + 0** ?

Theorem # **5:** $\underline{X + 0 = X}$

Step #4: Boolean Algebra

$$F_1 = AB + 0$$

Which Theorem can be applied to **AB + 0** ?

Theorem # **5:** $X + 0 = X$

AB + 0 can be simplified to **AB**

Step #4: Boolean Algebra

$$F_1 = \text{A B} + \text{0}$$

Which Theorem can be applied to **AB + 0** ?

Theorem # **5:** $\text{X} + 0 = \text{X}$

AB + 0 can be simplified to **AB**

AB

Example #1: Boolean Algebra

Simplify the following Boolean expression and note the Boolean theorem used at each step. Put the answer in SOP form.

$F_1 = A\,A\,B + C\,\overline{C}\,D$ is the same as F = **AB**

$$F_1 = A\,A\,B + C\,\overline{C}\,D$$

$$F_1 = A\,B + C\,\overline{C}\,D \quad \text{; Theorem \#3}$$

$$F_1 = A\,B + 0\,D \quad \text{; Theorem \#4}$$

$$F_1 = A\,B + 0 \quad \text{; Theorem \#1}$$

$$F_1 = A\,B \quad \text{; Theorem \#5}$$

$$F_1 = A\,B$$

Example #2: Boolean Algebra

Simplify the following Boolean expression and note the Boolean theorem used at each step. Put the answer in SOP form.

$$F_2 = B B \overline{C} + B \overline{C}\,\overline{C} + \overline{A} B 1 + A \overline{B} 0$$

Example #2: Boolean Algebra

$$F_2 = B B \overline{C} + B \overline{C}\,\overline{C} + \overline{A} B 1 + A \overline{B} 0$$

What Theorem(s) can be applied?

Example #2: Boolean Algebra

$$F_2 = B B \overline{C} + B \overline{C}\,\overline{C} + \overline{A} B 1 + A \overline{B} 0$$

What Theorem(s) can be applied?

<u>1, 2, 3, 5, & 7</u>

Example #2: Boolean Algebra

$$F_2 = B\,B\,\overline{C} + B\,\overline{C}\,\overline{C} + \overline{A}\,B\,1 + A\,\overline{B}\,0$$

$$F_2 = B\,B\,\overline{C} + B\,\overline{C}\,\overline{C} + \overline{A}\,B\,1 + A\,\overline{B}\,0$$

Example #2: Boolean Algebra

$$F_2 = B\,B\,\overline{C} + B\,\overline{C}\,\overline{C} + \overline{A}\,B\,1 + A\,\overline{B}\,0$$

$$F_2 = B\,B\,\overline{C} + B\,\overline{C}\,\overline{C} + \overline{A}\,B\,1 + A\,\overline{B}\,0$$ Apply Theorem #**<u>3</u>** **<u>TWICE</u>!**

Example #2: Boolean Algebra

$$F_2 = B B \overline{C} + B \overline{C}\,\overline{C} + \overline{A} B 1 + A \overline{B} 0$$

$$F_2 = B B \overline{C} + B \overline{C}\,\overline{C} + \overline{A} B 1 + A \overline{B} 0$$

Apply Theorem #3 **TWICE!**

$$F_2 = B \overline{C} + B \overline{C} + \overline{A} B 1 + A \overline{B} 0$$

Example #2: Boolean Algebra

$$F_2 = B\,B\,\overline{C} + B\,\overline{C}\,\overline{C} + \overline{A}\,B\,1 + A\,\overline{B}\,0$$

$F_2 = B\,B\,\overline{C} + B\,\overline{C}\,\overline{C} + \overline{A}\,B\,1 + A\,\overline{B}\,0$ Apply Theorem #3 **TWICE**!

$F_2 = B\,\overline{C} + B\,\overline{C} + \overline{A}\,B\,1 + A\,\overline{B}\,0$ Apply Theorem #**7**

Example #2: Boolean Algebra

$$F_2 = B\,B\,\overline{C} + B\,\overline{C}\,\overline{C} + \overline{A}\,B\,1 + A\,\overline{B}\,0$$

$$F_2 = B\,B\,\overline{C} + B\,\overline{C}\,\overline{C} + \overline{A}\,B\,1 + A\,\overline{B}\,0$$ Apply Theorem #3 **TWICE**!

$$F_2 = B\,\overline{C} + B\,\overline{C} + \overline{A}\,B\,1 + A\,\overline{B}\,0$$ Apply Theorem #7

$$F_2 = B\,\overline{C} + \overline{A}\,B\,1 + A\,\overline{B}\,0$$

Example #2: Boolean Algebra

$$F_2 = B B \overline{C} + B \overline{C}\,\overline{C} + \overline{A} B 1 + A \overline{B} 0$$

$$F_2 = B B \overline{C} + B \overline{C}\,\overline{C} + \overline{A} B 1 + A \overline{B} 0$$ Apply Theorem #3 **TWICE**!

$$F_2 = B \overline{C} + B \overline{C} + \overline{A} B 1 + A \overline{B} 0$$ Apply Theorem #7

$$F_2 = B \overline{C} + \overline{A} B 1 + A \overline{B} 0$$ Apply Theorem #**2**

Example #2: Boolean Algebra

$$F_2 = B B \overline{C} + B \overline{C}\,\overline{C} + \overline{A} B 1 + A \overline{B} 0$$

$F_2 = B B \overline{C} + B \overline{C}\,\overline{C} + \overline{A} B 1 + A \overline{B} 0$ Apply Theorem #3 **TWICE**!

$F_2 = B \overline{C} + B \overline{C} + \overline{A} B 1 + A \overline{B} 0$ Apply Theorem #7

$F_2 = B \overline{C} + \overline{A} B 1 + A \overline{B} 0$ Apply Theorem #2

$F_2 = B \overline{C} + \overline{A} B + A \overline{B} 0$

Example #2: Boolean Algebra

$$F_2 = B B \overline{C} + B \overline{C}\,\overline{C} + \overline{A} B 1 + A \overline{B} 0$$

$$F_2 = B B \overline{C} + B \overline{C}\,\overline{C} + \overline{A} B 1 + A \overline{B} 0$$ Apply Theorem #3 **TWICE**!

$$F_2 = B \overline{C} + B \overline{C} + \overline{A} B 1 + A \overline{B} 0$$ Apply Theorem #7

$$F_2 = B \overline{C} + \overline{A} B 1 + A \overline{B} 0$$ Apply Theorem #2

$$F_2 = B \overline{C} + \overline{A} B + A \overline{B} 0$$ Apply Theorem #**1**

Example #2: Boolean Algebra

$$F_2 = B B \overline{C} + B \overline{C}\,\overline{C} + \overline{A} B 1 + A \overline{B} 0$$

$F_2 = B B \overline{C} + B \overline{C}\,\overline{C} + \overline{A} B 1 + A \overline{B} 0$ Apply Theorem #3 **TWICE**!

$F_2 = B \overline{C} + B \overline{C} + \overline{A} B 1 + A \overline{B} 0$ Apply Theorem #7

$F_2 = B \overline{C} + \overline{A} B 1 + A \overline{B} 0$ Apply Theorem #2

$F_2 = B \overline{C} + \overline{A} B + A \overline{B} 0$ Apply Theorem #1

$F_2 = B \overline{C} + \overline{A} B + 0$

Example #2: Boolean Algebra

$$F_2 = B B \overline{C} + B \overline{C}\,\overline{C} + \overline{A} B 1 + A \overline{B} 0$$

$F_2 = B B \overline{C} + B \overline{C}\,\overline{C} + \overline{A} B 1 + A \overline{B} 0$ Apply Theorem #3 **TWICE**!

$F_2 = B \overline{C} + B \overline{C} + \overline{A} B 1 + A \overline{B} 0$ Apply Theorem #7

$F_2 = B \overline{C} + \overline{A} B 1 + A \overline{B} 0$ Apply Theorem #2

$F_2 = B \overline{C} + \overline{A} B + A \overline{B} 0$ Apply Theorem #1

$F_2 = B \overline{C} + \overline{A} B + 0$ Apply Theorem #**5**

Example #2: Boolean Algebra

$$F_2 = B B \overline{C} + B \overline{C}\,\overline{C} + \overline{A} B 1 + A \overline{B} 0$$

$F_2 = B B \overline{C} + B \overline{C}\,\overline{C} + \overline{A} B 1 + A \overline{B} 0$ Apply Theorem #3 **TWICE**!

$F_2 = B \overline{C} + B \overline{C} + \overline{A} B 1 + A \overline{B} 0$ Apply Theorem #7

$F_2 = B \overline{C} + \overline{A} B 1 + A \overline{B} 0$ Apply Theorem #2

$F_2 = B \overline{C} + \overline{A} B + A \overline{B} 0$ Apply Theorem #1

$F_2 = B \overline{C} + \overline{A} B + 0$ Apply Theorem #5

$F_2 = B \overline{C} + \overline{A} B$

Example #2: Boolean Algebra

$$F_2 = B B \overline{C} + B \overline{C} \, \overline{C} + \overline{A} B 1 + A \overline{B} 0$$

$F_2 = B B \overline{C} + B \overline{C} \, \overline{C} + \overline{A} B 1 + A \overline{B} 0$ Apply Theorem #3 **TWICE**!

$F_2 = B \overline{C} + B \overline{C} + \overline{A} B 1 + A \overline{B} 0$ Apply Theorem #7

$F_2 = B \overline{C} + \overline{A} B 1 + A \overline{B} 0$ Apply Theorem #2

$F_2 = B \overline{C} + \overline{A} B + A \overline{B} 0$ Apply Theorem #1

$F_2 = B \overline{C} + \overline{A} B + 0$ Apply Theorem #5

$F_2 = B \overline{C} + \overline{A} B$

$F_2 = B \overline{C} + \overline{A} B$

Example #2: Boolean Algebra

$$F_2 = B B \overline{C} + B \overline{C}\,\overline{C} + \overline{A} B 1 + A \overline{B} 0$$

can be simplified to… $F_2 = B\overline{C} + \overline{A}B$

Boolean Laws

Boolean Laws

Commutative Law

*Theorem #10A – **AND** Function*

$$X \cdot Y = Y \cdot X$$

*Theorem #10B – **OR** Function*

$$X + Y = Y + X$$

Boolean Laws

Associative Law

*Theorem #11A – **AND** Function*

$$X(YZ) = (XY)Z$$

*Theorem #11B – **OR** Function*

$$X + (Y + Z) = (X + Y) + Z$$

Boolean Laws

Distributive Law

*Theorem #12A – **AND** Function*

$$X(Y+Z) = XY + XZ$$

*Theorem #12B – **OR** Function*

$$(X+Y)(W+Z) = XW + XZ + YW + YZ$$

Boolean Laws

Distributive Law

When using Distributive Law, use the **FOIL** method!

FOIL Method

First – Outer – Inner – Last

Example #3: Boolean Algebra

Simplify the following Boolean expression and note the Boolean theorem used at each step. Put the answer in SOP form.

$$F_3 = \overline{R}\,T + (R + \overline{S})(\overline{R} + T)$$

Example #3: Boolean Algebra

$$F_3 = \overline{R}\,T + (R + \overline{S})(\overline{R} + T)$$

Solution

$$F_3 = \overline{R}\,T + \left(R + \overline{S}\right)\left(\overline{R} + T\right)$$

Example #3: Boolean Algebra

$$F_3 = \overline{R}\,T + (R + \overline{S})(\overline{R} + T)$$

Solution

$F_3 = \overline{R}\,T + \left(R + \overline{S}\right)\left(\overline{R} + T\right)$ Theorem #12B

Example #3: Boolean Algebra

$$F_3 = \overline{R}\,T + (R + \overline{S})(\overline{R} + T)$$

Solution

$$F_3 = \overline{R}\,T + \left(R + \overline{S}\right)\left(\overline{R} + T\right)$$

Theorem #12B

$$F_3 = \overline{R}\,T + R\,\overline{R} + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$$

Example #3: Boolean Algebra

$$F_3 = \overline{R}\,T + (R + \overline{S})(\overline{R} + T)$$

Solution

$$F_3 = \overline{R}\,T + \left(R + \overline{S}\right)\left(\overline{R} + T\right)$$ Theorem #12B

$$F_3 = \overline{R}\,T + R\,\overline{R} + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$$ Theorem #4

Example #3: Boolean Algebra

$$F_3 = \overline{R}\,T + (R + \overline{S})(\overline{R} + T)$$

Solution

$$F_3 = \overline{R}\,T + \left(R + \overline{S}\right)\left(\overline{R} + T\right)$$ Theorem #12B

$$F_3 = \overline{R}\,T + R\,\overline{R} + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$$ Theorem #4

$$F_3 = \overline{R}\,T + 0 + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$$

Example #3: Boolean Algebra

$$F_3 = \overline{R}\,T + (R + \overline{S})(\overline{R} + T)$$

Solution

$$F_3 = \overline{R}\,T + \left(R + \overline{S}\right)\left(\overline{R} + T\right)$$ Theorem #12B

$$F_3 = \overline{R}\,T + R\,\overline{R} + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$$ Theorem #4

$$F_3 = \overline{R}\,T + 0 + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$$ Theorem #5

Example #3: Boolean Algebra

$$F_3 = \overline{R}\,T + (R + \overline{S})(\overline{R} + T)$$

Solution

$F_3 = \overline{R}\,T + \left(R + \overline{S}\right)\left(\overline{R} + T\right)$ Theorem #12B

$F_3 = \overline{R}\,T + R\,\overline{R} + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$ Theorem #4

$F_3 = \overline{R}\,T + 0 + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$ Theorem #5

$F_3 = \overline{R}\,T + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$

Example #3: Boolean Algebra

$$F_3 = \overline{R}\,T + (R + \overline{S})(\overline{R} + T)$$

Solution

$F_3 = \overline{R}\,T + \left(R + \overline{S}\right)\left(\overline{R} + T\right)$ Theorem #12B

$F_3 = \overline{R}\,T + R\,\overline{R} + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$ Theorem #4

$F_3 = \overline{R}\,T + 0 + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$ Theorem #5

$F_3 = \overline{R}\,T + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$ Theorem #12A

Example #3: Boolean Algebra

$$F_3 = \overline{R}\,T + (R + \overline{S})(\overline{R} + T)$$

Solution

$$F_3 = \overline{R}\,T + \left(R + \overline{S}\right)\left(\overline{R} + T\right)$$ Theorem #12B

$$F_3 = \overline{R}\,T + R\,\overline{R} + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$$ Theorem #4

$$F_3 = \overline{R}\,T + 0 + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$$ Theorem #5

$$F_3 = \overline{R}\,T + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$$ Theorem #12A

$$F_3 = T\left(\overline{R} + R + \overline{S}\right) + \overline{S}\,\overline{R}$$

Example #3: Boolean Algebra

$$F_3 = \overline{R}\,T + (R + \overline{S})(\overline{R} + T)$$

Solution

$F_3 = \overline{R}\,T + \left(R + \overline{S}\right)\left(\overline{R} + T\right)$ — Theorem #12B

$F_3 = \overline{R}\,T + R\,\overline{R} + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$ — Theorem #4

$F_3 = \overline{R}\,T + 0 + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$ — Theorem #5

$F_3 = \overline{R}\,T + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$ — Theorem #12A

$F_3 = T\left(\overline{R} + R + \overline{S}\right) + \overline{S}\,\overline{R}$ — Theorem #8

Example #3: Boolean Algebra

$$F_3 = \overline{R}\,T + (R + \overline{S})(\overline{R} + T)$$

Solution

$$F_3 = \overline{R}\,T + \left(R + \overline{S}\right)\left(\overline{R} + T\right)$$ Theorem #12B

$$F_3 = \overline{R}\,T + R\,\overline{R} + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$$ Theorem #4

$$F_3 = \overline{R}\,T + 0 + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$$ Theorem #5

$$F_3 = \overline{R}\,T + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$$ Theorem #12A

$$F_3 = T\left(\overline{R} + R + \overline{S}\right) + \overline{S}\,\overline{R}$$ Theorem #8

$$F_3 = T\left(1 + \overline{S}\right) + \overline{S}\,\overline{R}$$

Example #3: Boolean Algebra

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Example #3: Boolean Algebra

$$F_3 = \overline{R}\,T + (R + \overline{S})(\overline{R} + T)$$

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$$F_3 = T\left(\overline{R} + R + \overline{S}\right) + \overline{S}\,\overline{R}$$ Theorem #8

$$F_3 = T\left(1 + \overline{S}\right) + \overline{S}\,\overline{R}$$ Theorem #6

$$F_3 = T(1) + \overline{S}\,\overline{R}$$

Example #3: Boolean Algebra

$$F_3 = \overline{R}\,T + (R + \overline{S})(\overline{R} + T)$$

Solution

$F_3 = \overline{R}\,T + \left(R + \overline{S}\right)\left(\overline{R} + T\right)$	Theorem #12B
$F_3 = \overline{R}\,T + R\,\overline{R} + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$	Theorem #4
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$F_3 = \overline{R}\,T + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$	Theorem #12A
$F_3 = T\left(\overline{R} + R + \overline{S}\right) + \overline{S}\,\overline{R}$	Theorem #8
$F_3 = T\left(1 + \overline{S}\right) + \overline{S}\,\overline{R}$	Theorem #6
$F_3 = T(1) + \overline{S}\,\overline{R}$	Theorem #2

Example #3: Boolean Algebra

$$F_3 = \overline{R}\,T + (R + \overline{S})(\overline{R} + T)$$

Solution

$F_3 = \overline{R}\,T + \left(R + \overline{S}\right)\left(\overline{R} + T\right)$	Theorem #12B
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$F_3 = \overline{R}\,T + R\,T + \overline{S}\,\overline{R} + \overline{S}\,T$	Theorem #12A
$F_3 = T\left(\overline{R} + R + \overline{S}\right) + \overline{S}\,\overline{R}$	Theorem #8
$F_3 = T\left(1 + \overline{S}\right) + \overline{S}\,\overline{R}$	Theorem #6
$F_3 = T(1) + \overline{S}\,\overline{R}$	Theorem #2
$\boxed{F_3 = T + \overline{S}\,\overline{R}}$	Final Answer

Boolean Consensus Theorems

Consensus Theorems

Theorem #13A

$$X + \overline{X}Y = X + Y$$

Theorem #13B

$$\overline{X} + XY = \overline{X} + Y$$

Theorem #13C

$$X + \overline{X}\,\overline{Y} = X + \overline{Y}$$

Theorem #13D

$$\overline{X} + X\overline{Y} = \overline{X} + \overline{Y}$$

Theorem #13E

$$X + XY = X$$

Example #4: Boolean Algebra

Simplify the following Boolean expression and note the Boolean theorem used at each step. Put the answer in SOP form.

$$F_4 = P S + P \overline{Q} \overline{S} + P Q S$$

Example #4: Boolean Algebra

$$F_4 = P S + P \overline{Q} \, \overline{S} + P Q S$$

$$F_4 = P S + P \overline{Q} \, \overline{S} + P Q S$$

Example #4: Boolean Algebra

$$F_4 = PS + P\overline{Q}\,\overline{S} + PQS$$

$$F_4 = PS + P\overline{Q}\,\overline{S} + PQS$$

Theorem #12A

Example #4: Boolean Algebra

$$F_4 = P\,S + P\,\overline{Q}\,\overline{S} + P\,Q\,S$$

$$F_4 = P\,S + P\,\overline{Q}\,\overline{S} + P\,Q\,S$$

Theorem #12A

$$F_4 = P\left(S + \overline{Q}\,\overline{S}\right) + P\,Q\,S$$

Example #4: Boolean Algebra

$$F_4 = P\,S + P\,\overline{Q}\,\overline{S} + P\,Q\,S$$

$$F_4 = P\,S + P\,\overline{Q}\,\overline{S} + P\,Q\,S$$ Theorem #12A

$$F_4 = P\left(S + \overline{Q}\,\overline{S}\right) + P\,Q\,S$$ Theorem #13C

Example #4: Boolean Algebra

$$F_4 = P\,S + P\,\overline{Q}\,\overline{S} + P\,Q\,S$$

$$F_4 = P\,S + P\,\overline{Q}\,\overline{S} + P\,Q\,S$$

Theorem #12A

$$F_4 = P\left(S + \overline{Q}\,\overline{S}\right) + P\,Q\,S$$

Theorem #13C

$$F_4 = P\left(S + \overline{Q}\right) + P\,Q\,S$$

Example #4: Boolean Algebra

$$F_4 = P\,S + P\,\overline{Q}\,\overline{S} + P\,Q\,S$$

$$F_4 = P\,S + P\,\overline{Q}\,\overline{S} + P\,Q\,S$$ Theorem #12A

$$F_4 = P\left(S + \overline{Q}\,\overline{S}\right) + P\,Q\,S$$ Theorem #13C

$$F_4 = P\left(S + \overline{Q}\right) + P\,Q\,S$$ Theorem #12A

Example #4: Boolean Algebra

$$F_4 = PS + P\overline{Q}\,\overline{S} + PQS$$

$$F_4 = PS + P\overline{Q}\,\overline{S} + PQS$$

Theorem #12A

$$F_4 = P\left(S + \overline{Q}\,\overline{S}\right) + PQS$$

Theorem #13C

$$F_4 = P\left(S + \overline{Q}\right) + PQS$$

Theorem #12A

$$F_4 = PS + P\overline{Q} + PQS$$

Example #4: Boolean Algebra

$$F_4 = P S + P \overline{Q} \, \overline{S} + P Q S$$

$$F_4 = P S + P \overline{Q} \, \overline{S} + P Q S$$ Theorem #12A

$$F_4 = P \left(S + \overline{Q} \, \overline{S} \right) + P Q S$$ Theorem #13C

$$F_4 = P \left(S + \overline{Q} \right) + P Q S$$ Theorem #12A

$$F_4 = P S + P\overline{Q} + P Q S$$ Theorem #12A

Example #4: Boolean Algebra

$$F_4 = P\,S + P\,\overline{Q}\,\overline{S} + P\,Q\,S$$

$$F_4 = P\,S + P\,\overline{Q}\,\overline{S} + P\,Q\,S$$ Theorem #12A

$$F_4 = P\left(S + \overline{Q}\,\overline{S}\right) + P\,Q\,S$$ Theorem #13C

$$F_4 = P\left(S + \overline{Q}\right) + P\,Q\,S$$ Theorem #12A

$$F_4 = P\,S + P\overline{Q} + P\,Q\,S$$ Theorem #12A

$$F_4 = PS\left(1 + Q\right) + P\,\overline{Q}$$

Example #4: Boolean Algebra

$$F_4 = P\,S + P\,\overline{Q}\,\overline{S} + P\,Q\,S$$

$F_4 = P\,S + P\,\overline{Q}\,\overline{S} + P\,Q\,S$	Theorem #12A
$F_4 = P\left(S + \overline{Q}\,\overline{S}\right) + P\,Q\,S$	Theorem #13C
$F_4 = P\left(S + \overline{Q}\right) + P\,Q\,S$	Theorem #12A
$F_4 = P\,S + P\overline{Q} + P\,Q\,S$	Theorem #12A
$F_4 = PS\left(1 + Q\right) + P\,\overline{Q}$	Theorem #6

Example #4: Boolean Algebra

$$F_4 = PS + P\overline{Q}\,\overline{S} + PQS$$

$$F_4 = PS + P\overline{Q}\,\overline{S} + PQS$$ Theorem #12A

$$F_4 = P\left(S + \overline{Q}\,\overline{S}\right) + PQS$$ Theorem #13C

$$F_4 = P\left(S + \overline{Q}\right) + PQS$$ Theorem #12A

$$F_4 = PS + P\overline{Q} + PQS$$ Theorem #12A

$$F_4 = PS\left(1 + Q\right) + P\overline{Q}$$ Theorem #6

$$F_4 = PS\left(1\right) + P\overline{Q}$$

Example #4: Boolean Algebra

$$F_4 = P\,S + P\,\overline{Q}\,\overline{S} + P\,Q\,S$$

$F_4 = P\,S + P\,\overline{Q}\,\overline{S} + P\,Q\,S$	Theorem #12A
$F_4 = P\left(S + \overline{Q}\,\overline{S}\right) + P\,Q\,S$	Theorem #13C
$F_4 = P\left(S + \overline{Q}\right) + P\,Q\,S$	Theorem #12A
$F_4 = P\,S + P\overline{Q} + P\,Q\,S$	Theorem #12A
$F_4 = PS\left(1 + Q\right) + P\,\overline{Q}$	Theorem #6
$F_4 = PS\left(1\right) + P\,\overline{Q}$	Theorem #2

Example #4: Boolean Algebra

$$F_4 = P\,S + P\,\overline{Q}\,\overline{S} + P\,Q\,S$$

$$F_4 = P\,S + P\,\overline{Q}\,\overline{S} + P\,Q\,S$$ Theorem #12A

$$F_4 = P\left(S + \overline{Q}\,\overline{S}\right) + P\,Q\,S$$ Theorem #13C

$$F_4 = P\left(S + \overline{Q}\right) + P\,Q\,S$$ Theorem #12A

$$F_4 = P\,S + P\overline{Q} + P\,Q\,S$$ Theorem #12A

$$F_4 = PS\left(1 + Q\right) + P\,\overline{Q}$$ Theorem #6

$$F_4 = PS\left(1\right) + P\,\overline{Q}$$ Theorem #2

$$\boxed{F_4 = PS + P\,\overline{Q}}$$

Example #4: Boolean Algebra

$$F_4 = P\,S + P\,\overline{Q}\,\overline{S} + P\,Q\,S$$

can be simplified to $\underline{F_4 = PS + P\,\overline{Q}}$

Summary (See Handout)

1) $X \cdot 0 = 0$

2) $X \cdot 1 = X$

3) $X \cdot X = X$

4) $X \cdot \overline{X} = 0$

5) $X + 0 = X$

6) $X + 1 = 1$

7) $X + X = X$

8) $X + \overline{X} = 1$

9) $\overline{\overline{X}} = X$

Boolean Theorems

Commutative Law:

10A) $X \cdot Y = Y \cdot X$

10B) $X + Y = Y + X$

Associative Law:

11A) $X(YZ) = (XY)Z$

11B) $X + (Y + Z) = (X + Y) + Z$

Distributive Law:

12A) $X(Y + Z) = XY + XZ$

12B) $(X + Y)(W + Z) = XW + XZ + YW + YZ$

Consensus Theorem:

13A) $X + \overline{X}Y = X + Y$

13B) $\overline{X} + XY = \overline{X} + Y$

13C) $X + \overline{X}\,\overline{Y} = X + \overline{Y}$

13D) $\overline{X} + X\overline{Y} = \overline{X} + \overline{Y}$

13E) $X + XY = X$