Warm-Up | Figures in the Coordinate Plane

Lesson Question

Lesson Goals

Calculate ________ of triangles and quadrilaterals.

Classify ________ and quadrilaterals.

Develop ________ proofs of quadrilaterals in the coordinate ________.

Use coordinate ________ to solve problems.

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Words to Know

Fill in this table as you work through the lesson. You may also use the glossary to help you.

verify	to apply what is known to ______ an expected result
coordinate	numbers in an ______ of numbers
distance	a measurement of ______ between two points
perimeter	the distance ______ a two-dimensional shape
slope	in a relation, the relationship between the change in the ______ value and the change in the ______ value

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Reviewing the Distance and Slope Formulas

Calculate the **distance** between the points with the **coordinates** $(-2, 3)$ and $(4, -1)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(4 - (-2))^2 + (-1 - 3)^2}$

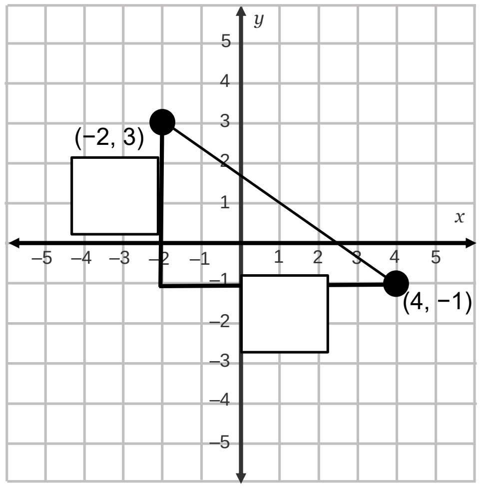
$= \sqrt{36 + 16}$

$= \sqrt{52}$

$= \sqrt{4 \cdot 13}$

$=$ ☐

Label the vertical and horizontal distances between the points.



What is the **slope** of the segment?

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-3}{4-(-2)}$

$= \frac{-4}{6}$

$=$ ☐

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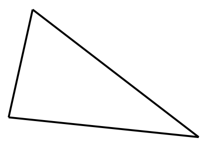
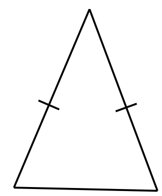
Classification of Triangles

Triangle classifications include scalene, isosceles, equilateral, and right.



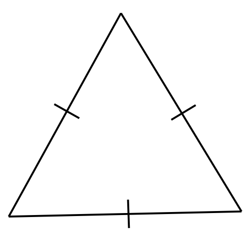
______

does not have any congruent sides



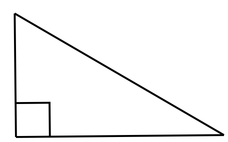
Isosceles

has at least ______ congruent sides



Equilateral

has three ______ sides



Right

a triangle with one ______ angle

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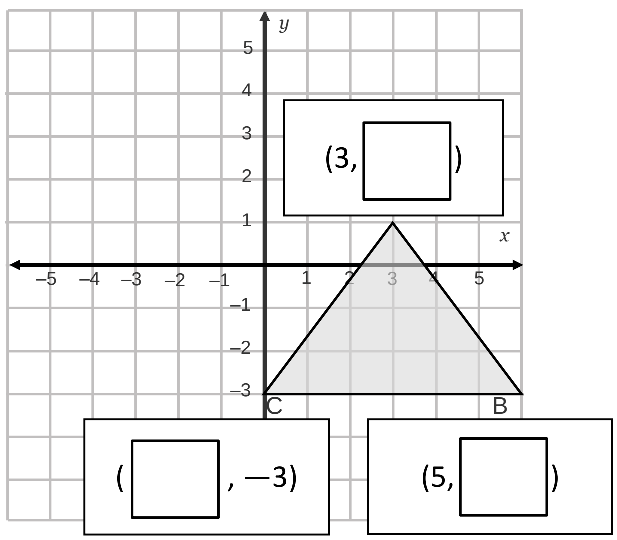
Classifying a Triangle in the Coordinate Plane

How can we use coordinates to classify triangles?



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{AC} = \sqrt{(3-0)^2 + (1-(-3))^2} = \sqrt{25} = \square$$

$$\text{AB} = \sqrt{(3-6)^2 + (1-(-3))^2} = \sqrt{25} = \square$$

$\overline{\text{CB}}$ is horizontal, so we can just count on the grid. It is 6 units long.

We have two congruent sides, and so this is an ________ triangle.

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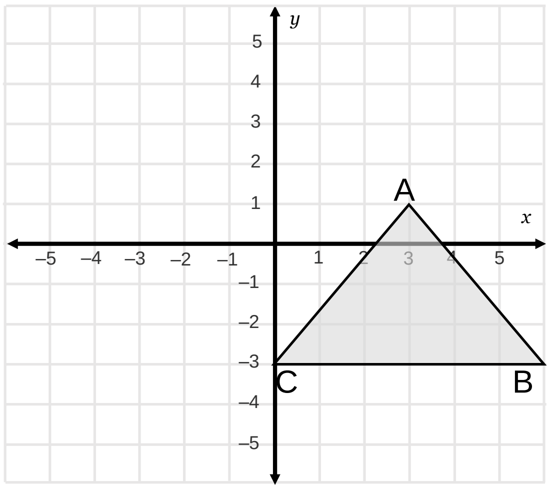
How can we use coordinates to classify triangles?

- We'll determine whether this is a right triangle by calculating the slopes to determine whether two sides are perpendicular.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{\overline{AC}} = \frac{1-(-3)}{3-0} = \square$$

$$m_{\overline{AB}} = \frac{1-(-3)}{3-6} = \square$$

Two nonvertical lines are ______ if and only if their slopes are opposite reciprocals. The slopes of $\overline{AC}$ and $\overline{AB}$ are not negative ______ of each other, so they are not perpendicular, and this isosceles triangle is not a ______ triangle.

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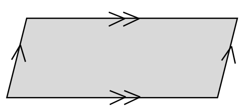
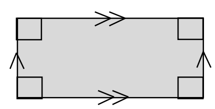
Classification of Quadrilaterals

- Quadrilateral classifications include parallelogram, rectangle, square, rhombus, trapezoid, and kite.



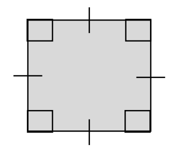
Parallelogram

a quadrilateral with both pairs of opposite sides ________



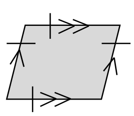
Rectangle

all of the angles are ________ angles



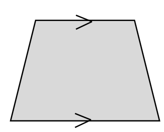
Square

a parallelogram with ________ right angles and all four ________ congruent



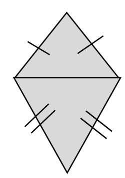
Rhombus

a parallelogram where all the sides are ________



Trapezoid

a quadrilateral with only one pair of sides being parallel



Kite

a quadrilateral whose two ________ sides are congruent and whose opposite sides are not congruent

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Classifying a Quadrilateral in the Coordinate Plane

Classify quadrilateral ABCD.

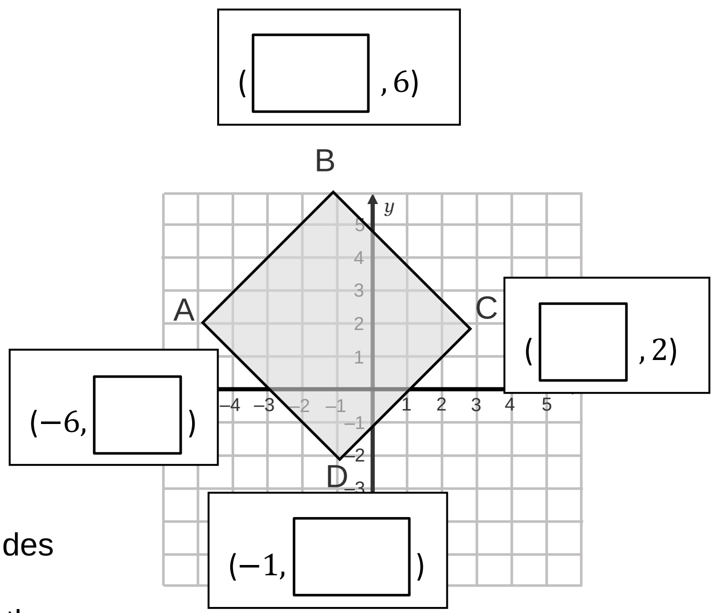
Input the missing coordinates.

$m_{\overline{AB}} = \frac{6-2}{-1-(-5)} = \frac{4}{4} = \square$

$m_{\overline{BC}} = \frac{2-6}{3-(-1)} = \frac{-4}{4} = \square$

$m_{\overline{CD}} = \frac{-2-2}{-1-3} = \frac{-4}{-4} = \square$

$m_{\overline{AD}} = \frac{-2-2}{-1-(-5)} = \frac{-4}{4} = \square$



Because the slopes of adjacent sides are negative reciprocals of each other, we can conclude that they form four ☐ angles.

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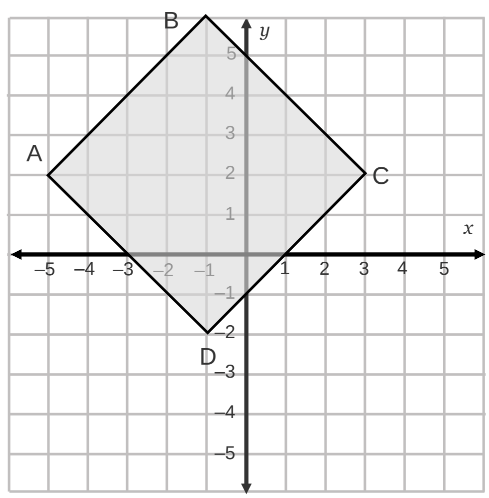
Classify quadrilateral ABCD.

AB = $\sqrt{(-5-(-1))^2+(2-6)^2}$

= $\sqrt{16+16}$

= ☐ units



BC = $\sqrt{(-1-3)^2+(6-2)^2}$ = $\sqrt{32}$

CD = $\sqrt{(3-(-1))^2+(2-(-2))^2}$ = ☐

AD = $\sqrt{(-5-(-1))^2+(2-(-2))^2}$ = $\sqrt{32}$

Because all four lengths are the same, we can conclude that this parallelogram is a ☐.

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Calculating the Perimeter of a Quadrilateral in the Coordinate Plane

Calculate the **perimeter** of quadrilateral $ABCD$.

Perimeter = $AB + BC + CD + AD$

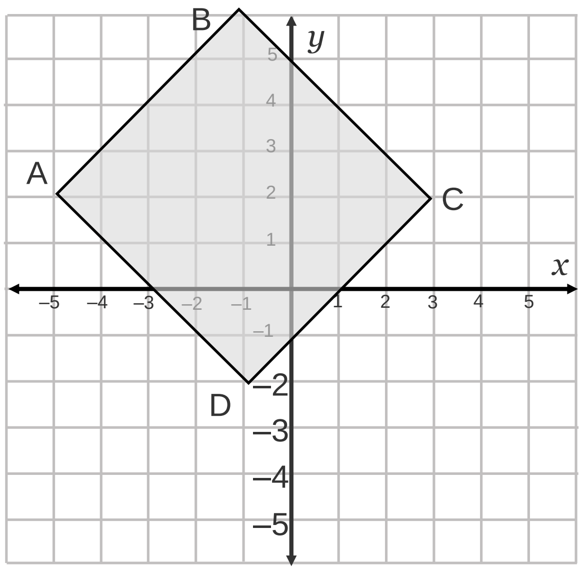
P = $\sqrt{32} + \sqrt{32} + \sqrt{32} + \sqrt{32}$

P = [ ]

P = $4 \cdot \sqrt{16 \cdot 2}$

P = $4 \cdot 4 \cdot \sqrt{2}$

P = $16 \cdot$ [ ]



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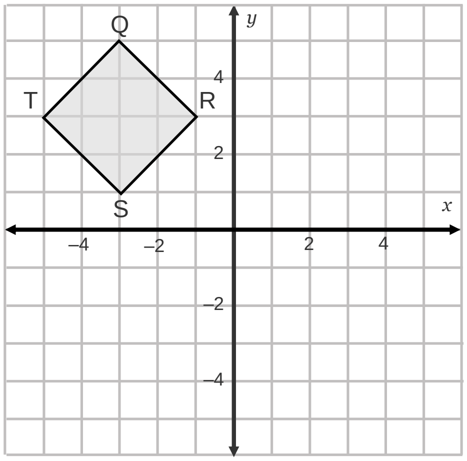
Proofs on the Coordinate Plane

Prove that the diagonals of square $QRST$ with vertices $(-3, 5)$, $(-1, 3)$, $(-3, 1)$, and $(-5, 3)$ bisect each other.



Find the midpoint of each diagonal.

$M_{\overline{QS}} = \left(\frac{-3+(-3)}{2}, \frac{5+1}{2}\right) = (\square, 3)$

$M_{\overline{RT}} = \left(\frac{-3+(-3)}{2}, \frac{5+1}{2}\right) = (-3, \square)$

The diagonals have the same midpoint, therefore, they $\square$ each other.

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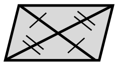
Using Theorems to Prove Classification of Special Quadrilaterals

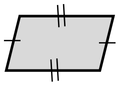
Theorems can be used to prove the classification of special quadrilaterals in the coordinate plane.

Parallelogram

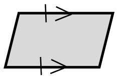
- Converse of parallelogram diagonal theorem
  - If the diagonals of a quadrilateral [ ] each other, then the quadrilateral is a parallelogram.



- Converse of parallelogram side theorem
  - If both pairs of opposite sides of a quadrilateral are [ ], then the quadrilateral is a parallelogram



- Single opposite side pair theorem
  - If you have opposite sides that are congruent [ ], then the quadrilateral is a parallelogram.



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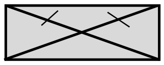
Rectangle

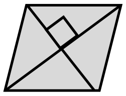
- Rectangle diagonal theorem
  - A parallelogram is a rectangle if and only if its ______ are congruent.



- Rectangle angle theorem
  - We only need one ______ angle to prove that the parallelogram is a rectangle.

Rhombus

- Rhombus diagonal theorem
  - A parallelogram is a ______ if and only if its diagonals are ______, or meet at a right angle.



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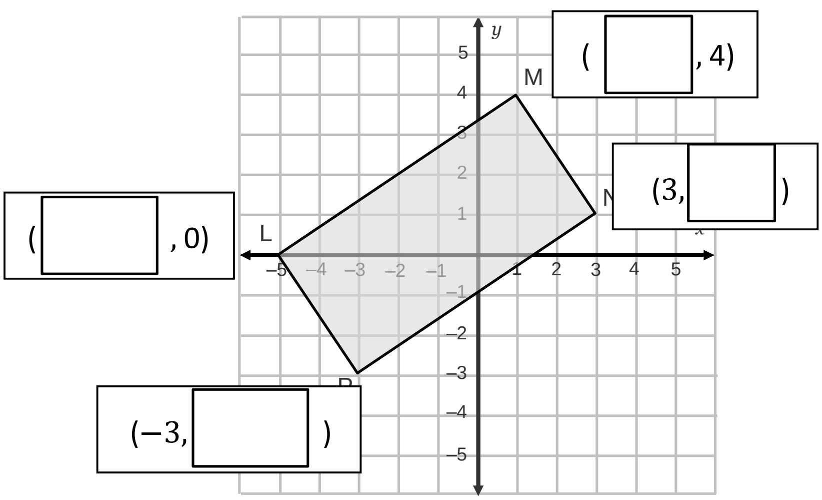
Proving Classification of Quadrilaterals in the Coordinate Plane: Distance Formula

Verify that the quadrilateral is a rectangle.

Step 1: Prove the quadrilateral is a parallelogram.

Let's use the converse of the parallelogram side theorem.

This theorem states that if opposite sides are ______, it is a parallelogram.



$LM = \sqrt{(-5-1)^2+(0-4)^2} = \sqrt{52}$

$PN = \sqrt{(-3-3)^2+(-3-1)^2} =$ ______

$LP = \sqrt{(-5-(-3))^2+(0-(-3))^2} =$ ______

$MN = \sqrt{(1-3)^2+(4-1)^2} = \sqrt{13}$

By the converse of the parallelogram side theorem:

opposite sides are congruent; therefore, it is a ______.

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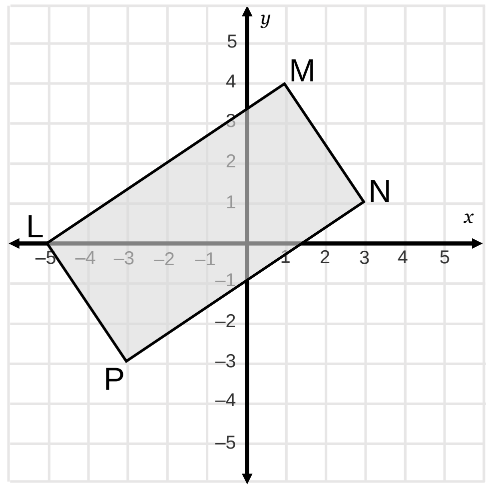
Proving Classification of Quadrilaterals in the Coordinate Plane

Prove that the quadrilateral is a rectangle.



Step 2: Prove that the parallelogram is a rectangle.

- The rectangle angle theorem states that a parallelogram is a rectangle if it has one [ ] angle.

Slope $\overline{LP} = \frac{0-(-3)}{-5-(-3)} =$ [ ]

Slope $\overline{NP} = \frac{1-(-3)}{3-(-3)} = \frac{4}{6} =$ [ ]

They are opposite reciprocals of each other, so the sides are [ ], which means that $\angle LPN$ is a right angle by the definition of perpendicular. Since the parallelogram has one right angle, then it is a [ ].

Summary | Figures in the Coordinate Plane

Lesson Question

How can coordinate algebra be used to verify or prove geometric properties?

Answer

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Review: Key Concepts

- Distance formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- ______ formula: $\frac{y_2 - y_1}{x_2 - x_1}$
- Midpoint formula: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Summary | Figures in the Coordinate Plane

Use this space to write any questions or thoughts about this lesson.