Pre-Cal

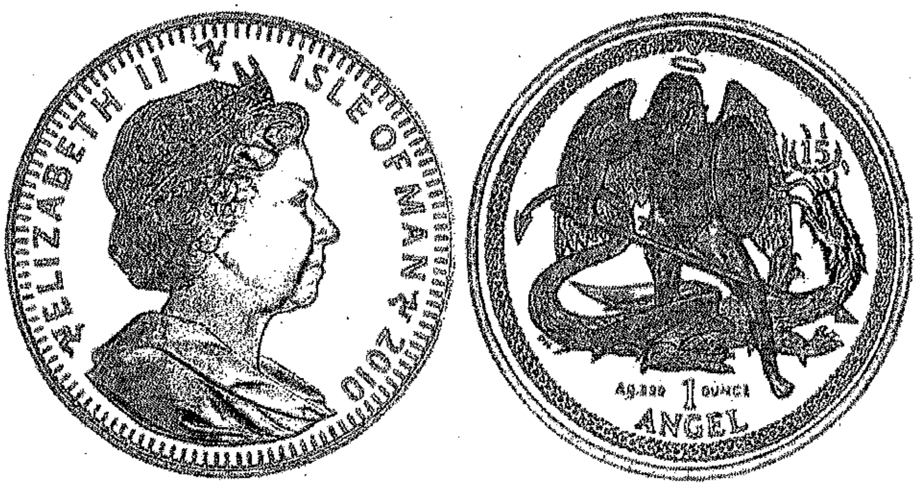
Day 1

Standards	**PS.SPCR.8** Use permutations and combinations to **solve** mathematical and real-world problems, including **determining** probabilities of compound events. **Justify** the results.
Learning Targets/I Can Statements	I can count how many outcomes an event may have using the fundamental Counting Principle, permutations or combinations.
Essential Question(s)	**How can we base decisions on chance? How can probability be used to simulate events and to predict future happenings?**
Resources	https://www.khanacademy.org/math/statistics-probability/counting-permutations-and-combinations https://www.mathplayground.com/JakeVSAstro_Archive08.html
Learning Activities or Experiences	1st: Recall questions (attached) 2nd: Counting, permutations, and combinations a. Counting Principle and Factorial b. Permutations Alternative: Counting Principle and Factorial 3rd: Interactive Math Playground Counting Principle Activity (link above) 4th: Assignment

How Many Choices Do I Have

Fundamental Counting Principle

When you estimate how likely an event is to occur, you need to know the sample space. The sample space is the set of all possible outcomes in a random experiment. An event is a subset of the sample space. For instance, the outcome of flipping a fair coin once is either Head or Tail, so the sample space is the set {H, T}. There are only two outcomes that are possible.



How can you determine the size of the sample space when the experiment is more complicated and involves more than two outcomes?

Ex: Thomas wanted to purchase a pizza for dinner. He went online to Pizza Hut. Thomas was overwhelmed with all the different choices. Help Thomas determine how many different choices of pizzas are available. The chart is below.

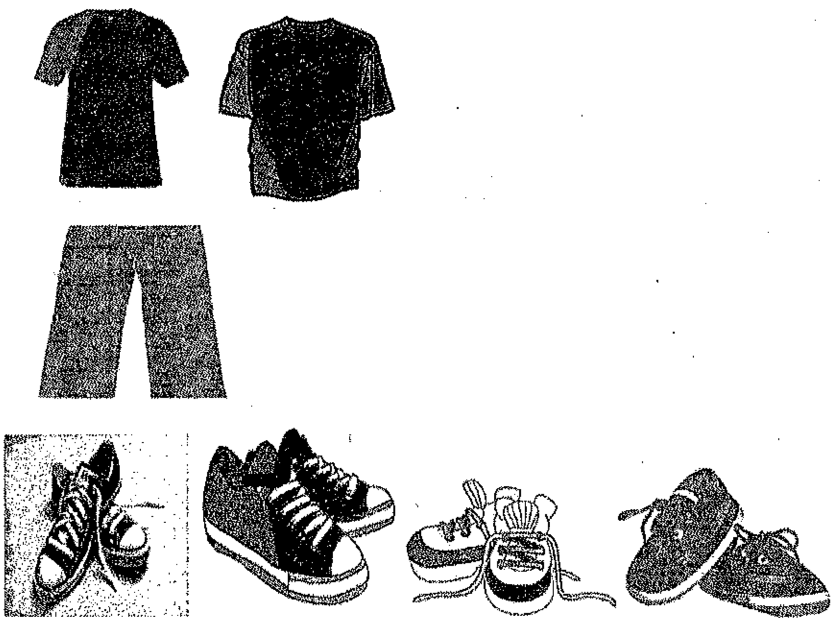
Crust	Sauce	Toppings
Thin	Alfredo	Sausage
Hand-Tossed	Barbecue	Pepperoni
Stuffed	Marinara	Cheese
Pan		Vegetables
Deep-Dished		Chicken
		All Meat

Fundamental Counting: If there are **x** outcomes for one event, followed by **y** outcomes for another event, there are a total of xy outcomes.

Simple Terms: Multiply the different events

5 Crust * 3 Sauce * 6 Toppings = 5* 3 * 6 = 90

Your Turn: Determine how many different outfits you can make.



Your Turn: Miss Crawford wants to buy a new car. She is considering either a hybrid or an electric car. She can choose a standard or a luxury model. There are red, silver, black, and white colors available to choose from. How many possible choices does Miss Crawford have if she chooses one item from each group?

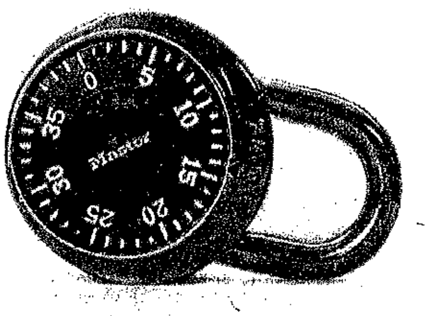
a. 8
b. 16
c. 6
d. 12

Permutations

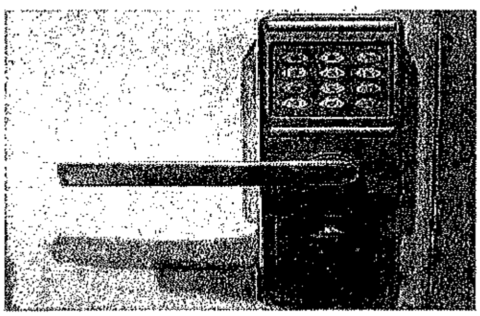
When you set up a numerical password, you can arrange the digits in many different ways, and maybe repeat some of the numbers. The number of ways a general password is generated could be mind boggling depending on how you set it up. The number of digits used, with or without letters, numbers, and characters, with or without repetition, all increase the possible passwords you could create.

Permutation - a possible order or arrangement of objects chosen from a set of objects

Factorial - the product of a sequence of whole numbers from to 1 in descending order



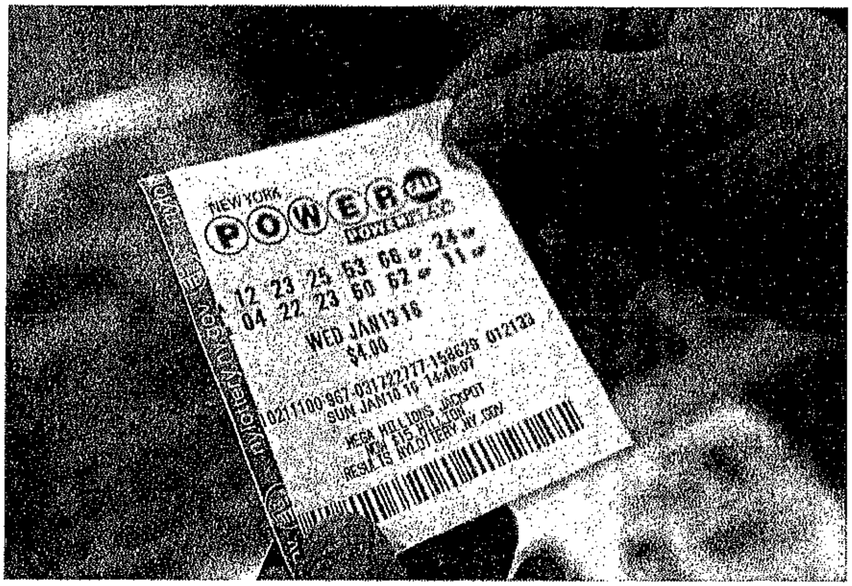
Before you further explore the rules, consider a simple 4-digit password with just the numbers from 0 to 9.



To solve: 10 x 10 x 10 x 10 = 10,000

Explanation: Since there are 10 digits from 0-9 and there is a total of 4 digits for the combination. We have to multiply ten six times.

Your turn: Jack wanted to figure out how many possible lottery numbers they are for the Pick 3. You can only use 0 -9 for each of the 3 digits.



a. 10 x 9 x 8 = 720
b. 10 x 10 x 10 = 1000
c. 10 x 3 = 30
d. 10 + 10 + 10 = 30

Extension for learning: Each digit uses numbers from 0-99. What will be the number of possible outcomes for the Pick 3?

Name:
Class:
Date:

Question #1

A restaurant serves 3 kinds of pasta, 5 kinds of sauces, and 4 side dishes. The number of different meals that include 1 pasta, 1 sauce, and 1 side dish is $3 \times 5 \times 4 = 60$. If they stop serving one type of side dish, how many different meals are possible?

A) 11

B) 24

C) 45

D) 59

Question #2

A local restaurant advertises 3-topping pizzas at a discount price. With 6 toppings to choose from, how many 3-topping pizzas can be made if all the toppings are different? Note: Pepperoni (P), Cheese (C), and Sausage (S) is the same as CSP or SPC.

A) 15

B) 18

C) 20

D) 120

Question #3

A lock company advertises that their new combination lock has "nearly an infinite number of combinations possible." The lock requires a three letter code, with 26 possible choices for each of the three letters. Which expression represents the correct calculation for the number of possible lock combinations?

A) $26 \times 25 \times 24 \times \ldots \times 3 \times 2 \times 1$

B) $26 \times 26 \times 26$

C) $26 \times 25 \times 24$

D) $\frac{26\times25\times24}{3\times2\times1}$

Question #4

Ms. Hampton's sixth grade class is going on a field trip to the art museum. The museum requires students to tour in groups of three. If there are 24 students in Ms. Hampton's class, how many different combinations of three-student groups are possible?

A) 69

B) 72

C) 12,144

D) 13,824

Question #5

Gino brings home one book from each of his 5 school subjects and places them side by side on a bookshelf. In how many different orders can Gino arrange the 5 books on the shelf where Science is always first?

A) 15

B) 24

C) 120

D) 625

Pre-Cal
Day 2

Standards	*A1.ACE.1** ***Create** and **solve** equations and inequalities in one variable that model real-world problems involving linear, quadratic, simple rational, and exponential relationships. Interpret the solutions and determine whether they are reasonable. (Limit to ~~linear; quadratic~~; exponential with integer exponents.*
Learning Targets/I Can Statements	I can solve exponential growth problems. I can solve exponential decay problems. I can solve exponential equations.
Essential Question(s)	**What real life experiences represent exponential functions?** **How do you determine if different experiences apply to exponential growth or decay?**
Resources	https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:exponential-growth-decay https://www.mathwarehouse.com/exponential-growth/exponential-models-in-real-world.php
Learning Activities or Experiences	1st: Recall questions (attached) 2nd: Watch the Khan Academy video (link above) a. Introduction to exponential functions b. Exponential vs Linear c. Exponential Growth d. Exponential Decay Alternative: Notes on Exponential functions, Growth & Decay 3rd: Growth in the Real World (link above) 4th: Assignment

Recall Questions

1. Simplify: the exponential expression
   $(2x^3y^2)^5$

2. What is 6 raised to the 3rd power?

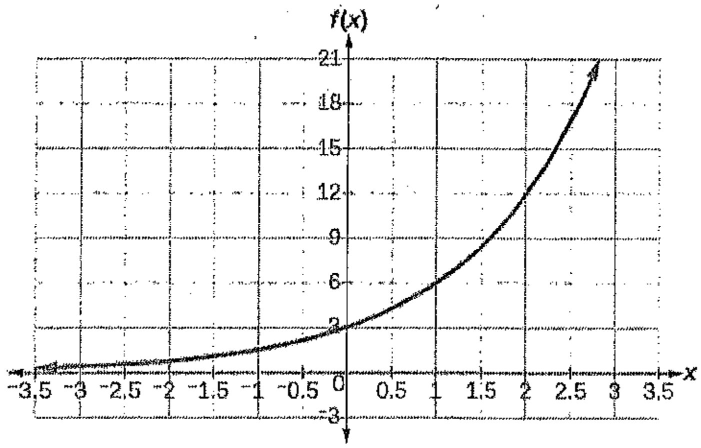
3. Simplify: $\frac{y^{17}}{y^5}$

4. Simplify: $(x^3)(x^5)$

EXPONENTIAL FUNCTIONS

Exponential functions look somewhat similar to functions you have seen before, in that they involve exponents, but there is a big difference, in that the variable is now the power, rather than the base.

Ex: $y = 4^x$



***Exponential functions do not pass through the x intercept. They are curved lines.

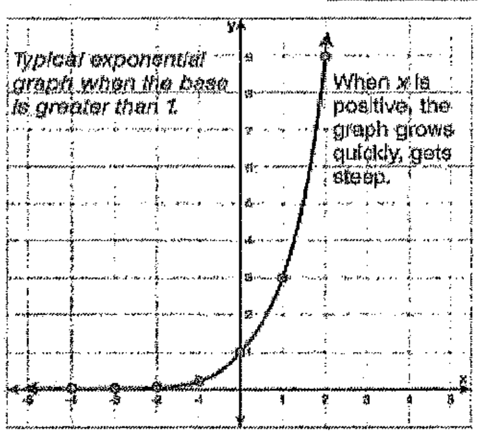
Example #1:

Evaluate and graph the exponential function $y = 3^x$ for $-3 \le x \le 3$.

Solution:

Make a table to show ordered pairs that satisfy the equation $y = 3^x$. Then, graph the ordered pairs.

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27



Exponential Growth and Decay

Exponential growth – is the manner in which a quantity grows over a time. It occurs when the instantaneous rate of change of a quantity with respect to time is proportional to the quantity itself.

Exponential growth formula: $y = a(1 + r)^t$

a = initial amount   r = growth rate (decimal)   t = time

b = (1+r) growth factor

For exponential growth, the growth factor will always be greater than 1.

Always change rate from a percent to a decimal.

Ex: Thomas purchased a baseball card for 1.25 in 1980. The value of it increased by 3% each year. What is the card worth today?

$Y = 1.25(1 + .03)^{40}$

$Y = 1.25(1.03)^{40}$

= $4.08

Explanation: Change 3% to .03. Then substitute values in to formula. Type the entire formula in the calculator and solve.

Your Turn:

Jennifer worked for the Census Bureau. Columbia had 99, 000 residents in 2010. The population increased by 2% per year. What will be the population of Columbia in 2025?

Exponential Decay– is the manner in which a quantity decreases over a time. It occurs when the instantaneous rate of change of a quantity with respect to time is proportional to the quantity itself.

Exponential decay formula: $y = a(1 - r)^t$

a = initial amount   r = growth rate (decimal)   t = time

b = (1 - r) growth factor

For exponential decay, the growth factor will always be less than 1.

Always change rate from a percent to a decimal.

Ex: There was infestation of roaches at Mr. Ham's house. He called the exterminator to kill the roaches. There were approximately 6250 roaches in the house. After he fumigated the house, the roaches died at a rate of 15 percent per day. How many roaches were left after 13 days?

$Y = 6250(1 - .15)^{13}$

$Y = 6250(.85)^{13}$

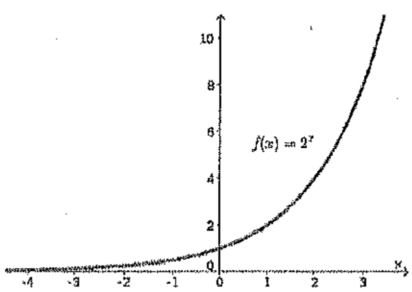
$=756$

Your Turn:

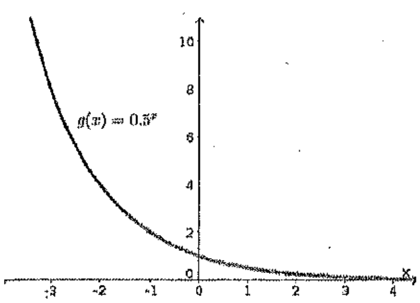
Tasha's mother purchased a car in 2013. The car cost $23500. The value of the car depreciated by 7 percent each year. What is the value of the car today?

Difference of Exponential Growth and Decay Graphs

Exponential Growth Graph below: Positive number greater than 1



Exponential Decay Graph below: Positive number less than 1



Compare and Contrast both graphs:

What do you notice?

What are similarities between positive and negative linear equations?

Name:
Class:
Date:

Question #1

A radioactive substance decays at a rate of 25% every 10 years. Which equation represents the amount of the substance (*S*) remaining from 100 grams after 300 years?

A $S = 100(0.25)^{300/10}$

B $S = 300(0.25)^{100/10}$

C $S = 100(0.75)^{300/10}$

D $S = 300(0.75)^{100/10}$

Question #2

Bushra purchases a car for $12,900. The car will depreciate at a rate of 15% each year.

After how many years will the value of the car be less than $3,000?

A 6 years

B 7 years

C 8 years

D 9 years

Question #3

Marianne bought a car at a price of $18,000. The price of the car depreciated at a constant rate of r% per year. The price of the car after 2 years was $13,005.

Which equation can be used to find the rate of depreciation?

A $13{,}005 = 18{,}000(1 + r)^2$

B $13{,}005 = 18{,}000(1 - r)^2$

C $18{,}000 = 13{,}005(1 + r)^2$

D $18{,}000 = 13{,}005(1 - r)^2$

Question #4

The population of a certain town is 158,260 and increases exponentially at the rate of 6% every year.

Which equation best represents the population after x years?

A $y = 158{,}260\ (0.06)^x$

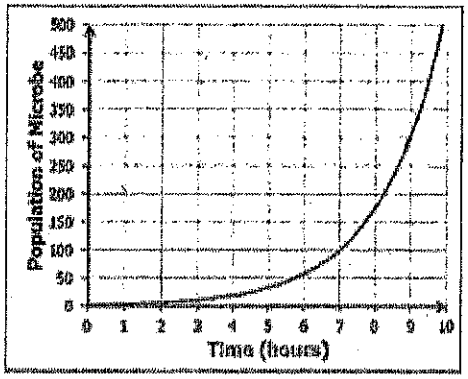
B $y = 158{,}260\ (0.94)^x$

C $y = 158{,}260\ (1.06)^x$

D $y = 158{,}260\ (1.6)^x$

Question #5

A microbiologist is studying a microbe population and finds that the population growth follows the exponential model shown in the graph.



What is the approximate population after 9 hours?

A 50

B 100

C 300

D 500

Name:
Class:
Date:

Question #1

An economist predicts that the number of employees with a certain company will increase by 50% each year. There are 600 employees now. According to the economist's prediction, how many employees will be with the company exactly 3 years from now?

A) 900

B) 1350

C) 1500

D) 2025

Question #2

A botanist predicts that the height of a certain tree will increase by 2% every year. If the height of the tree is now 50 feet, what is its predicted height 2 years from now?

A) 50.04 feet

B) 51 feet

C) 52 feet

D) 52.02 feet

Question #3

A biologist predicts that the height of a certain tree will increase exponentially with time, tripling every 60 years. The tree is now 5 meters tall. According to the biologist's prediction, in how many years would the tree become 45 meters tall?

A) 120

B) 180

C) 540

D) 800

Question #4

Mr. Andrews is saving to buy a violin that costs $1,000. He has already saved $450 and decides to put all of this money into a new savings account. The money in this account will earn 6% interest, which is compounded quarterly. Which model would be appropriate to determine how many years, *y*, Mr. Andrews will have to wait until he has earned enough money in this account to buy the violin?

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A **= amount of money accumulated after *n* years, including interest, *P* = principal amount (the initial amount deposited), *r* = annual rate of interest, *n* = number of times the interest is compounded per year, and *t* = number of years the amount is deposited**

A) $1{,}000 = 450\left(\frac{1.06}{4}\right)^{4y}$

B) $450 = 1{,}000\left(\frac{1.06}{4}\right)^{y}$

C) $450 = 1{,}000\left(1 + \frac{0.06}{4}\right)^{4y}$

D) $1{,}000 = 450\left(1 + \frac{0.06}{4}\right)^{4y}$

Question #5

The present value of an antique is $200. The value of the antique 4 years ago was $189.50 and increases every year by a constant factor, p.

Which equation represents the scenario *correctly*?

A $200(4)^{p} = 189.50$

B $200(p)^{4} = 189.50$

C $189.50(4)^{p} = 200$

D $189.50(p)^{4} = 200$

Day 3
Pre-Calculus

Standards	**PC.AREI.7** Solve a simple system consisting of a linear equation and a quadratic equation in two or three variables algebraically and graphically. Understand that such systems may have zero, one, two, or infinitely many solutions.
Learning Targets/I Can Statements	I can determine the intersection point of equations from a graph. I can understand that systems of equations have one, zero, or infinite solutions.
Essential Question(s)	**How I determine the intersection point of two equations by graphing?** **How can I determine the number of solutions when given a system of equations?**
Resources	https://www.khanacademy.org/math/algebra-basics/alg-basics-systems-of-equations http://crctlessons.com/systems-of-equations-game.html https://www.desmos.com/calculator https://www.khanacademy.org/math/algebra-basics/alg-basics-systems-of-equations
Learning Activities or Experiences	1st: Recall questions (attached) 2nd: Watch the Khan Academy video (link above) system of linear equation basics and number of solutions to systems of equations Alternative: Notes on systems on equations 3rd: System of linear equations game 4th: Assignment

Recall Questions

1. How many solutions are there for the following equation 3x + 8 = 6x -3?

2. How many solutions are there for the following equation 2(x + 3) = 5X -3X + 3?

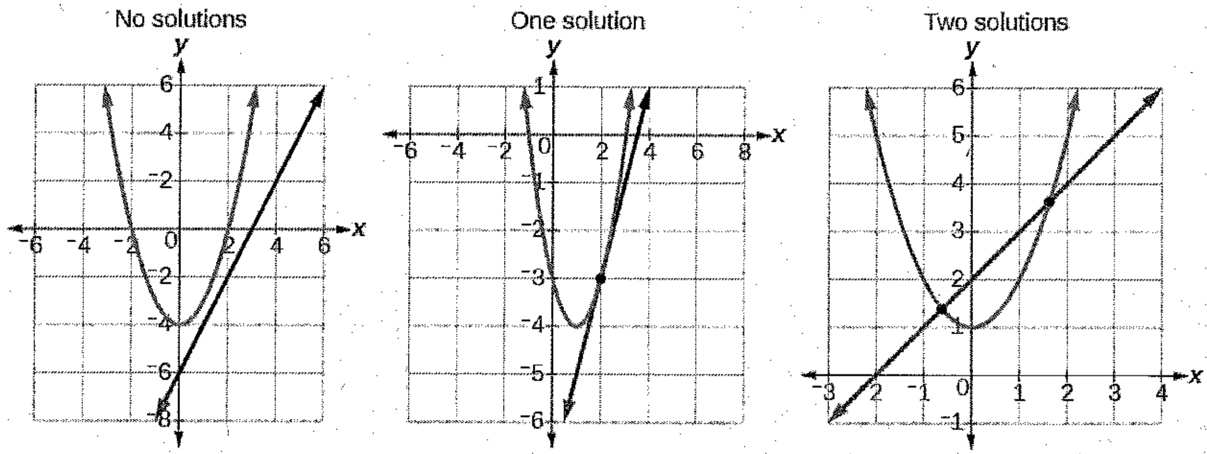
3. How many solutions are there for the following equation $\frac{4X-6}{2}$ = 2X - 3?

4. What is the solution: -3x +7x +4 = 4x –10

5. What is the solution: $\frac{2x-3}{4}=\frac{3x+1}{3}$

Systems of Equations

(Quadratics and Linear)



(a) (b) (c)

System of Linear Equations: is a collection of two or more equations.

<u>Number of solutions</u>

We will be looking at two ways to find the number of solutions to a system of linear equations.

1st : We will be looking at graphs of systems of equations.

2nd: We will be looking at linear equations and quadratics:

<u>One solution</u>

There is one solution when the graphs intersect at a given point.

<u>No solution</u>

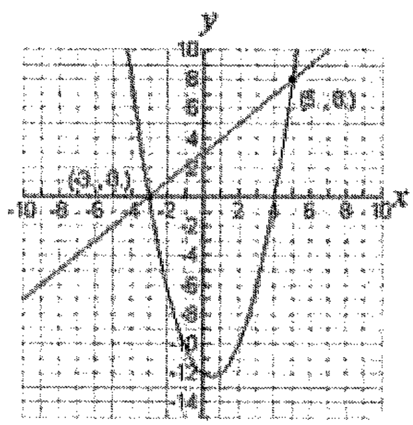
There is no solution when the quadratic and the linear equation never touch.

Two solutions

When only one line and the parabola touch and two points.

Solutions of system of equations by graphing

The intersections of equations is the solution.



The solution to the above system of equations is (-3,0) and (5,8).

System of equations

Algebraically

The coordinate that satisfies both equations is the solution to the system of equations.

Ex: $y = x^2 + x - 2$ and $y = -x + 1$ (-3, 4) (1,0)

$4 = (-3)^2 + (-3) - 2$ $(4) = -(-3)+1$

$4 = 4$ $4 = 4$

$0 = 1^2 + (1) - 2$ $0 = -(1) + 1$

$0 = 0$ $0 = 0$

Pre-Cal
Day 4

Standards	**G.GGPE.1*** Understand that the standard equation of a circle is derived from the definition of a circle and the distance formula. **G.GGPE.7*** Use the distance and midpoint formulas to determine distance and midpoint in a coordinate plane, as well as areas of triangles and rectangles, when given coordinates.
Learning Targets/I Can Statements	I can recognize conic sections as intersections of planes and cones.
Essential Questions:	How to recognize conic sections as intersections of planes and cones?
Resources	https://www.virtualnerd.com/algebra-2/conic-sections/identifying/conics/definition https://www.virtualnerd.com/algebra-2/conic-sections/midpoint-distance-formulas/distance-formula/distance-formula-definition
Learning Activities or Experiences	1st : Recall Questions: **Warm Up** **Find the slope of the line that connects each pair of points.** **1.** (5, 7) and (−1, 6) **2.** (3, −4) and (−4, 3) **Find the distance between each pair of points.** **3.** (−2, 12) and (6, −3) **4.** (1, 5) and (4, 1) 2nd: **What are Conic Sections?** (attached) 3rd: **What is the Distance Formula?** (attached) 4th: Assignment: Introduction to Conic Sections (attached)

What Is the Distance Formula?

What is the distance formula?

Virtual Nerd™

www.VirtualNerd.com

SUMMARY

The lines labeled 'x' and 'y' are the x and y-axes

We use subscripts '1' and '2' to distinguish between the two points

The distance, or length, of the line segment is represented by the variable 'd'

The symbol over the '$(x_2-x_1)^2+(y_2-y_1)^2$' is the square root sign

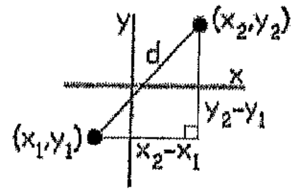
The thin red line represents: 'x_2-x_1'

The thin purple line represents: 'y_2-y_1'

The square in the corner of the triangle means 90 degrees

Distance Formula

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$



NOTES

Part 1) Definition

The distance formula is used to find the distance between two points

For this problem, think of distance as the length of a line segment connecting two points

Those two points are called the 'endpoints' of a line segment

Part 2) The Formula

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

There is a formula to find the distance, or length, of a line segment

This distance is represented by the variable 'd'

Each point has an x and y-coordinate

We use subscripts 1 and 2 to distinguish between the two points

The symbol over the '$(x_2-x_1)^2+(y_2-y_1)^2$' is the square root sign

The exponent '2' means 'squared'

Plug the coordinates into the right side of the formula to find the length of the line segment

Remember, 'length' here is the same as the distance

The 'coordinates' are the ordered pairs we're finding the distance between

So here the coordinates, or ordered pairs, would be (x_1, y_1) and (x_2, y_2)

Part 3) Trick for remembering the Distance Formula

Add two lines to our graphed line segment to make a right triangle

Make a horizontal line to represent the x-distance, shown on our diagram in red

Add in a vertical line to represent the y-distance, shown on our diagram in purple

These two lines intersect to form a right angle

The horizontal line segment will be the distance between our two x-coordinates:

$$x_2-x_1$$

The x-coordinates are 'x_1' and 'x_2'

We've represented 'x_2-x_1' with a red horizontal line on the graph

The vertical line segment will be the distance between our two y-coordinates:

$y_2 - y_1$

The y-coordinates are 'y_1' and 'y_2'

We've represented 'y_2-y_1' with a purple vertical line on the graph

---

We have the distance of the two sides we added, and d is the distance of the longest side

The longest side of a right triangle is called the hypotenuse

'd' is a variable representing 'distance'

Since we have a right triangle, we can use the Pythagorean Theorem:

$$\text{hypotenuse}^2 = \text{side}^2 + \text{side}^2$$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

In a right triangle, one angle is 90 degrees and the longest side is the hypotenuse

The exponent '2' means 'squared'

Remember, the length of the red line is represented by 'x_2-x_1'

Similarly, the length of the purple line is represented by 'y_2-y_1'

'd' is the hypotenuse since it's the longest side

Take the square root of each side to get the distance formula!

$$\sqrt{d^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\downarrow$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The symbol over all the variables is the square root sign

The exponent '2' means 'squared'

Taking the square root of anything that is already squared will get rid of the exponent '2'

We keep the square root on the right so we can plug our red and purple sides right in!

Taking the square root of 'd^2' won't give you '±d' because you can't have a negative distance

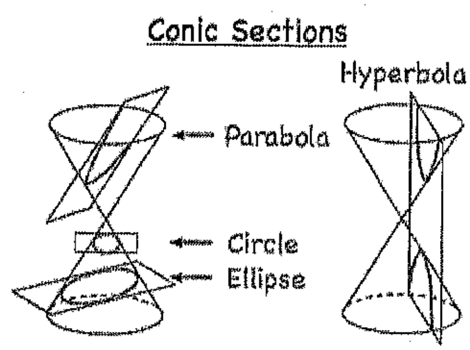
Since 'd' can only be positive, the other side of the equation will also be positive!

What are Conic Sections?

What are conic sections?

VirtualNerd
www.VirtualNerd.com

SUMMARY



NOTES

You've probably seen conic sections like circles or parabolas before

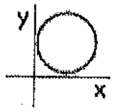
To see an example of a conic section, draw the intersection of a plane and a double cone

Part 1) Definition: conic section

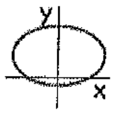
A CONIC SECTION is the curve created by the intersection of a plane and a double cone

Part 2) Types of curves

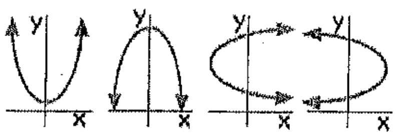
Intersecting a plane parallel to the bases of the double cone gives us a CIRCLE:



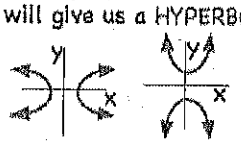
If the plane is at an angle and intersects just one cone, we get an ELLIPSE:



If the plane intersects a side and a base of a cone to create a U shape, we get a PARABOLA:



Intersecting the double cone with a plane that's perpendicular to its bases will give us a HYPERBOLA:



The upward- or downward-facing parabolas are the only conic sections that are functions

Part 3) Equations

A conic section will have an equation that looks like this:
$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$

In $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$
- A, B, C, D, E, and F are constants
- A, B, and C cannot all be 0

Part 4) Examples

Example 1:

$25x^2-4y^2-250x-16y+709=0$

$25x^2-4y^2-250x-16y+709=0$ follows the form of a conic section

Since there is no xy term the constant B is 0, but A and C are not

$25x^2-4y^2-250x-16y+709=0$ can be rewritten as:

$$\frac{(y+2)^2}{25}-\frac{(x-5)^2}{4}=1$$

This is a hyperbola

Example 2:

$-3y^2-x+12y-11=0$

$-3y^2-x+12y-11=0$ follows the form for a conic section

Since there are no x^2 and xy terms, the constants A and B are 0, but the constant C is not

$-3y^2-x+12y-11=0$ can be rewritten in the general form of a parabola:

$$x=-3(y-2)^2+1$$