Name:____________________

Honors Calculus
Unit 4 Review

Directions: This serves as a review for your Unit 4 Test—please note that this test covers material from sections 3.1-3.7. I recommend that you attempt/answer all these questions as they are similar to the ones on your Unit 4 Test. In addition, please review your quizzes from this unit as well as homework assignments and notes completed in class. On the test, you will need a #2 pencil and you will be able to use a calculator.

For each problem, find all points of absolute minima and maxima on the given interval.

1) $y = -2x^2 + 4x$; $[-1, 2]$

A) Absolute minimum: $(-1, -6)$
Absolute maximum: $(1, 2)$

B) No absolute minima.
No absolute maxima.

C) Absolute minimum: $(1, 2)$
Absolute maximum: $(2, 0)$

D) Absolute minimum: $(1, 2)$
No absolute maxima.

2) $y = -x^4 + x^2 + 4$; $[-1, 1]$

A) Absolute minimum: $(0, 4)$
Absolute maximum: $\left(\frac{\sqrt{2}}{2}, \frac{17}{4}\right)$

B) Absolute minimum: $\left(\frac{\sqrt{2}}{2}, \frac{17}{4}\right)$
Absolute maximum: $(0, 4)$

C) Absolute minima: $(-1, 4), (1, 4), (0, 4)$
Absolute maxima: $\left(-\frac{\sqrt{2}}{2}, \frac{17}{4}\right), \left(\frac{\sqrt{2}}{2}, \frac{17}{4}\right)$

D) No absolute minima.
No absolute maxima.

For each problem, find all points of relative minima and maxima.

3) $y = x^3 - 3x^2 + 1$

A) Relative minimum: $(4, 17)$
Relative maximum: $(8, 321)$

B) Relative minimum: $(2, -3)$
Relative maximum: $(0, 1)$

C) No relative minima.
No relative maxima.

D) No relative minima.
Relative maxima: $\left(\frac{1}{3}, \frac{19}{27}\right), \left(\frac{2}{3}, -\frac{1}{27}\right)$

4) $y = -(6x + 18)^{\frac{2}{3}}$

A) Relative minimum: $(-12, -9\sqrt[3]{4})$
No relative maxima.

B) No relative minima.
Relative maximum: $(-1, -2\sqrt[3]{18})$

C) No relative minima.
No relative maxima.

D) No relative minima.
Relative maximum: $(-3, 0)$

For each problem, find the open intervals where the function is increasing and decreasing.

5) $y = x^3 - 4x^2 + 5$

A) Increasing: $(-\infty, 0), \left(\frac{8}{3}, \infty\right)$ Decreasing: $\left(0, \frac{8}{3}\right)$

B) Increasing: $\left(0, \frac{8}{3}\right)$ Decreasing: $(-\infty, 0), \left(\frac{8}{3}, \infty\right)$

C) Increasing: $\left(\frac{1}{3}, \frac{8}{9}\right)$ Decreasing: $\left(-\infty, \frac{1}{3}\right), \left(\frac{8}{9}, \infty\right)$

D) Increasing: $(-\infty, 4), \left(\frac{32}{3}, \infty\right)$ Decreasing: $\left(4, \frac{32}{3}\right)$

6) $y = -x^3 + 3x^2$

A) Increasing: $(4, 8)$ Decreasing: $(-\infty, 4), (8, \infty)$

B) Increasing: $(0, 2)$ Decreasing: $(-\infty, 0), (2, \infty)$

C) Increasing: $(-\infty, 0), (2, \infty)$ Decreasing: $(0, 2)$

D) Increasing: $\left(-\infty, \frac{1}{3}\right), \left(\frac{2}{3}, \infty\right)$ Decreasing: $\left(\frac{1}{3}, \frac{2}{3}\right)$

For each problem, find the open intervals where the function is concave up and concave down.

7) $y = -x^5 + 2x^3 - 3$

A) Concave up: $\left(-\frac{\sqrt{15}}{5}, 0\right), \left(\frac{\sqrt{15}}{5}, \infty\right)$ Concave down: $\left(-\infty, -\frac{\sqrt{15}}{5}\right), \left(0, \frac{\sqrt{15}}{5}\right)$

B) Concave up: $\left(-\infty, -\frac{4\sqrt{15}}{5}\right), \left(4, \frac{4\sqrt{15}}{5}\right)$ Concave down: $\left(-\frac{4\sqrt{15}}{5}, 4\right), \left(\frac{4\sqrt{15}}{5}, \infty\right)$

C) Concave up: $\left(-\infty, -\frac{\sqrt{15}}{5}\right), \left(0, \frac{\sqrt{15}}{5}\right)$ Concave down: $\left(-\frac{\sqrt{15}}{5}, 0\right), \left(\frac{\sqrt{15}}{5}, \infty\right)$

D) Concave up: $\left(-\frac{\sqrt{15}}{15}, \frac{1}{3}\right), \left(\frac{\sqrt{15}}{15}, \infty\right)$ Concave down: $\left(-\infty, -\frac{\sqrt{15}}{15}\right), \left(\frac{1}{3}, \frac{\sqrt{15}}{15}\right)$

8) $y = -\frac{x^2}{4x + 4}$

A) Concave up: $\left(-\frac{1}{3}, \infty\right)$ Concave down: $\left(-\infty, -\frac{1}{3}\right)$
B) Concave up: $(-\infty, -4)$ Concave down: $(-4, \infty)$
C) Concave up: $(-\infty, -1)$ Concave down: $(-1, \infty)$
D) Concave up: $(-1, \infty)$ Concave down: $(-\infty, -1)$

Solve each optimization problem.

9) A company has started selling a new type of smartphone at the price of $\$160 - 0.1x$ where x is the number of smartphones manufactured per day. The parts for each smartphone cost \$70 and the labor and overhead for running the plant cost \$5000 per day. How many smartphones should the company manufacture and sell per day to maximize profit?

A) 450 B) 650 C) 550 D) 500

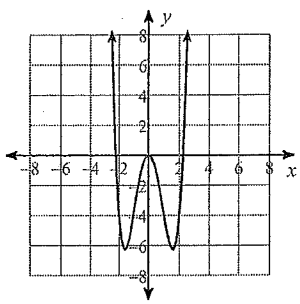
10) A farmer wants to construct a rectangular pigpen using 300 ft of fencing. The pen will be built next to an existing stone wall, so only three sides of fencing need to be constructed to enclose the pen. What dimensions should the farmer use to construct the pen with the largest possible area?

A) 76 ft (perpendicular to wall) by 148 ft (parallel to wall)
B) 77 ft (perpendicular to wall) by 146 ft (parallel to wall)
C) 80 ft (perpendicular to wall) by 140 ft (parallel to wall)
D) 75 ft (perpendicular to wall) by 150 ft (parallel to wall)

For each problem, find the: x and y intercepts, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

11) $y = -\frac{x^4}{8} + \frac{x^2}{8}$

A)



x-intercepts at $x = -\sqrt{5}, 0, \sqrt{5}$ y-intercept at $y = 0$

Critical points at: $x = -\frac{\sqrt{10}}{2}, 0, \frac{\sqrt{10}}{2}$

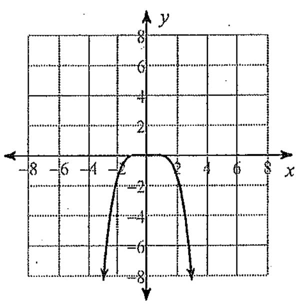
Increasing: $\left(-\frac{\sqrt{10}}{2}, 0\right), \left(\frac{\sqrt{10}}{2}, \infty\right)$ Decreasing: $\left(-\infty, -\frac{\sqrt{10}}{2}\right), \left(0, \frac{\sqrt{10}}{2}\right)$

Inflection points at: $x = -\frac{\sqrt{30}}{6}, \frac{\sqrt{30}}{6}$

Concave up: $\left(-\infty, -\frac{\sqrt{30}}{6}\right), \left(\frac{\sqrt{30}}{6}, \infty\right)$ Concave down: $\left(-\frac{\sqrt{30}}{6}, \frac{\sqrt{30}}{6}\right)$

Relative minima: $\left(-\frac{\sqrt{10}}{2}, -\frac{25}{4}\right), \left(\frac{\sqrt{10}}{2}, -\frac{25}{4}\right)$ Relative maximum: $(0, 0)$

B)



x-intercepts at $x = -1, 0, 1$ y-intercept at $y = 0$

Critical points at: $x = -\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}$

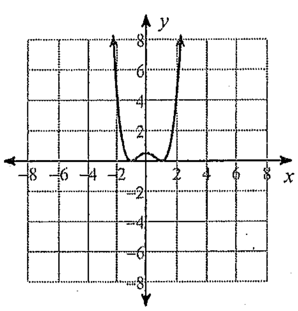
Increasing: $\left(-\infty, -\frac{\sqrt{2}}{2}\right), \left(0, \frac{\sqrt{2}}{2}\right)$ Decreasing: $\left(-\frac{\sqrt{2}}{2}, 0\right), \left(\frac{\sqrt{2}}{2}, \infty\right)$

Inflection points at: $x = -\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}$

Concave up: $\left(-\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}\right)$ Concave down: $\left(-\infty, -\frac{\sqrt{6}}{6}\right), \left(\frac{\sqrt{6}}{6}, \infty\right)$

Relative minimum: $(0, 0)$ Relative maxima: $\left(-\frac{\sqrt{2}}{2}, \frac{1}{32}\right), \left(\frac{\sqrt{2}}{2}, \frac{1}{32}\right)$

C)



x-intercepts at $x = -1, 1$ y-intercept at $y = \frac{1}{2}$

Critical points at: $x = -1, 0, 1$

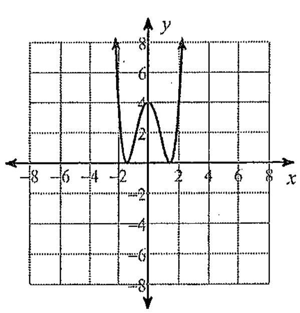
Increasing: $(-1, 0), (1, \infty)$ Decreasing: $(-\infty, -1), (0, 1)$

Inflection points at: $x = -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}$

Concave up: $\left(-\infty, -\frac{\sqrt{3}}{3}\right), \left(\frac{\sqrt{3}}{3}, \infty\right)$ Concave down: $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$

Relative minima: $(-1, 0), (1, 0)$ Relative maximum: $\left(0, \frac{1}{2}\right)$

D)



x-intercepts at $x = -\sqrt{2}, \sqrt{2}$ y-intercept at $y = 4$

Critical points at: $x = -\sqrt{2}, 0, \sqrt{2}$

Increasing: $(-\sqrt{2}, 0), (\sqrt{2}, \infty)$ Decreasing: $(-\infty, -\sqrt{2}), (0, \sqrt{2})$

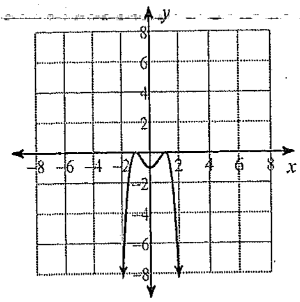
Inflection points at: $x = -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}$

Concave up: $\left(-\infty, -\frac{\sqrt{6}}{3}\right), \left(\frac{\sqrt{6}}{3}, \infty\right)$ Concave down: $\left(-\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}\right)$

Relative minima: $(-\sqrt{2}, 0), (\sqrt{2}, 0)$ Relative maximum: $(0, 4)$

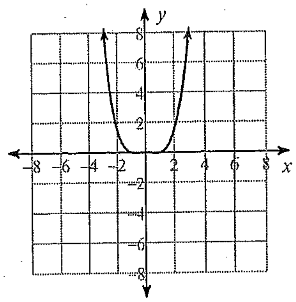
12) $y = -x^4 + 2x^2 - 1$

A)



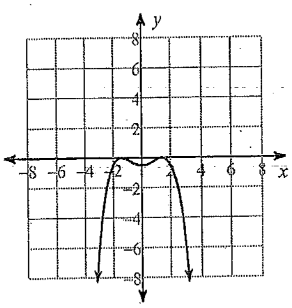
x-intercepts at $x = -1, 1$ y-intercept at $y = -1$
Critical points at: $x = -1, 0, 1$
Increasing: $(-\infty, -1), (0, 1)$ Decreasing: $(-1, 0), (1, \infty)$
Inflection points at: $x = -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}$
Concave up: $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$ Concave down: $\left(-\infty, -\frac{\sqrt{3}}{3}\right), \left(\frac{\sqrt{3}}{3}, \infty\right)$
Relative minimum: $(0, -1)$ Relative maxima: $(-1, 0), (1, 0)$

B)



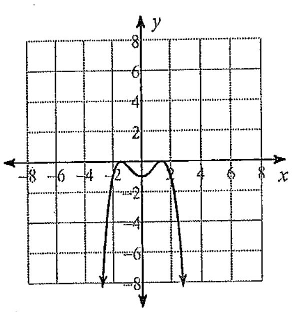
x-intercepts at $x = -1, 0, 1$ y-intercept at $y = 0$
Critical points at: $x = -\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}$
Increasing: $\left(-\frac{\sqrt{2}}{2}, 0\right), \left(\frac{\sqrt{2}}{2}, \infty\right)$ Decreasing: $\left(-\infty, -\frac{\sqrt{2}}{2}\right), \left(0, \frac{\sqrt{2}}{2}\right)$
Inflection points at: $x = -\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}$
Concave up: $\left(-\infty, -\frac{\sqrt{6}}{6}\right), \left(\frac{\sqrt{6}}{6}, \infty\right)$ Concave down: $\left(-\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}\right)$
Relative minima: $\left(-\frac{\sqrt{2}}{2}, -\frac{1}{32}\right), \left(\frac{\sqrt{2}}{2}, -\frac{1}{32}\right)$ Relative maximum: $(0, 0)$

C)



x-intercepts at $x = -\sqrt{2}, \sqrt{2}$ y-intercept at $y = -\frac{1}{2}$
Critical points at: $x = -\sqrt{2}, 0, \sqrt{2}$
Increasing: $(-\infty, -\sqrt{2}), (0, \sqrt{2})$ Decreasing: $(-\sqrt{2}, 0), (\sqrt{2}, \infty)$
Inflection points at: $x = -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}$
Concave up: $\left(-\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}\right)$ Concave down: $\left(-\infty, -\frac{\sqrt{6}}{3}\right), \left(\frac{\sqrt{6}}{3}, \infty\right)$
Relative minimum: $\left(0, -\frac{1}{2}\right)$ Relative maxima: $(-\sqrt{2}, 0), (\sqrt{2}, 0)$

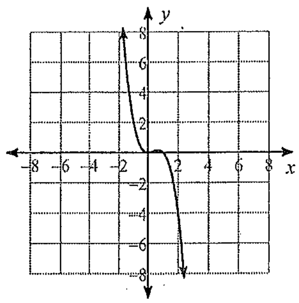
D)



x-intercepts at $x = -\sqrt{2}, \sqrt{2}$ y-intercept at $y = -1$
Critical points at: $x = -\sqrt{2}, 0, \sqrt{2}$
Increasing: $(-\infty, -\sqrt{2}), (0, \sqrt{2})$ Decreasing: $(-\sqrt{2}, 0), (\sqrt{2}, \infty)$
Inflection points at: $x = -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}$
Concave up: $\left(-\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}\right)$ Concave down: $\left(-\infty, -\frac{\sqrt{6}}{3}\right), \left(\frac{\sqrt{6}}{3}, \infty\right)$
Relative minimum: $(0, -1)$ Relative maxima: $(-\sqrt{2}, 0), (\sqrt{2}, 0)$

13) $y = -x^3 + x^2$

A)



x-intercepts at $x = 0, 1$ y-intercept at $y = 0$

Critical points at: $x = 0, \frac{2}{3}$

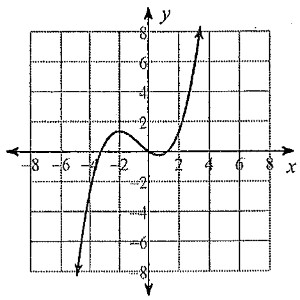
Increasing: $\left(0, \frac{2}{3}\right)$ Decreasing: $(-\infty, 0), \left(\frac{2}{3}, \infty\right)$

Inflection point at: $x = \frac{1}{3}$

Concave up: $\left(-\infty, \frac{1}{3}\right)$ Concave down: $\left(\frac{1}{3}, \infty\right)$

Relative minimum: $(0, 0)$ Relative maximum: $\left(\frac{2}{3}, \frac{4}{27}\right)$

B)



x-intercepts at $x = -1 - \sqrt{5}, 0, -1 + \sqrt{5}$ y-intercept at $y = 0$

Critical points at: $x = -2, \frac{2}{3}$

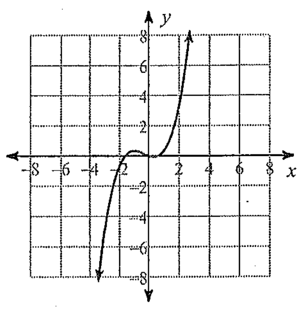
Increasing: $(-\infty, -2), \left(\frac{2}{3}, \infty\right)$ Decreasing: $\left(-2, \frac{2}{3}\right)$

Inflection point at: $x = -\frac{2}{3}$

Concave up: $\left(-\frac{2}{3}, \infty\right)$ Concave down: $\left(-\infty, -\frac{2}{3}\right)$

Relative minimum: $\left(\frac{2}{3}, -\frac{20}{81}\right)$ Relative maximum: $\left(-2, \frac{4}{3}\right)$

C)



x-intercepts at $x = \frac{-1 - \sqrt{5}}{2}, 0, \frac{-1 + \sqrt{5}}{2}$ y-intercept at $y = 0$

Critical points at: $x = -1, \frac{1}{3}$

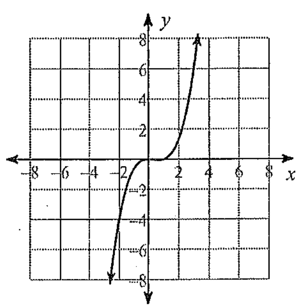
Increasing: $(-\infty, -1), \left(\frac{1}{3}, \infty\right)$ Decreasing: $\left(-1, \frac{1}{3}\right)$

Inflection point at: $x = -\frac{1}{3}$

Concave up: $\left(-\frac{1}{3}, \infty\right)$ Concave down: $\left(-\infty, -\frac{1}{3}\right)$

Relative minimum: $\left(\frac{1}{3}, -\frac{5}{81}\right)$ Relative maximum: $\left(-1, \frac{1}{3}\right)$

D)



x-intercepts at $x = 0, 1$ y-intercept at $y = 0$

Critical points at: $x = 0, \frac{2}{3}$

Increasing: $(-\infty, 0), \left(\frac{2}{3}, \infty\right)$ Decreasing: $\left(0, \frac{2}{3}\right)$

Inflection point at: $x = \frac{1}{3}$

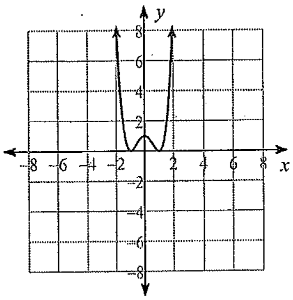
Concave up: $\left(\frac{1}{3}, \infty\right)$ Concave down: $\left(-\infty, \frac{1}{3}\right)$

Relative minimum: $\left(\frac{2}{3}, -\frac{4}{81}\right)$ Relative maximum: $(0, 0)$

For each problem, find the: x and y intercepts, asymptotes, x-coordinates of the critical points, and x-coordinates of the inflection points. Using this information, sketch the graph of the function.

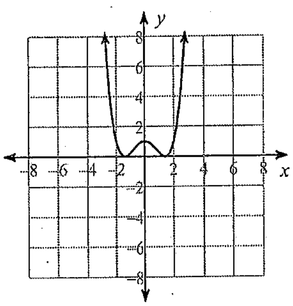
14) $y = -\frac{x^4}{4} + x^2 - 1$

A)



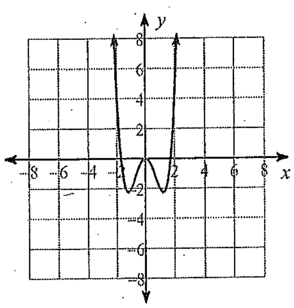
x-intercepts at $x = -1, 1$ y-intercept at $y = 1$
No vertical asymptotes exist.
No horizontal asymptotes exist.
Critical points at: $x = -1, 0, 1$
Inflection points at: $x = -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}$

B)



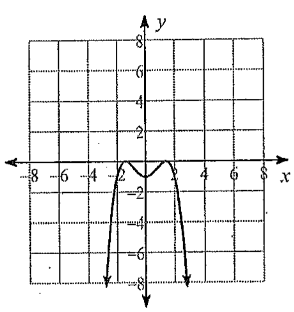
x-intercepts at $x = -\sqrt{2}, \sqrt{2}$ y-intercept at $y = 1$
No vertical asymptotes exist.
No horizontal asymptotes exist.
Critical points at: $x = -\sqrt{2}, 0, \sqrt{2}$
Inflection points at: $x = -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}$

C)



x-intercepts at $x = -\sqrt{3}, 0, \sqrt{3}$ y-intercept at $y = 0$
No vertical asymptotes exist.
No horizontal asymptotes exist.
Critical points at: $x = -\frac{\sqrt{6}}{2}, 0, \frac{\sqrt{6}}{2}$
Inflection points at: $x = -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}$

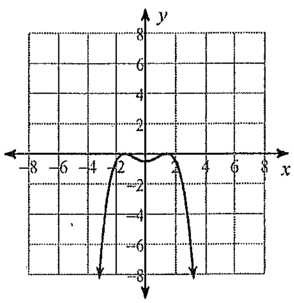
D)



x-intercepts at $x = -\sqrt{2}, \sqrt{2}$ y-intercept at $y = -1$
No vertical asymptotes exist.
No horizontal asymptotes exist.
Critical points at: $x = -\sqrt{2}, 0, \sqrt{2}$
Inflection points at: $x = -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}$

15) $y = \frac{x^4}{8} - \frac{5x^2}{8}$

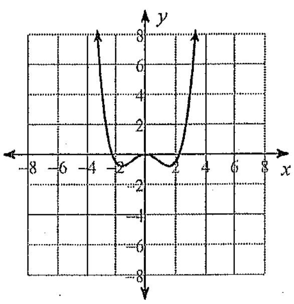
A)



x-intercepts at $x = -\sqrt{2}, \sqrt{2}$ y-intercept at $y = -\frac{1}{2}$

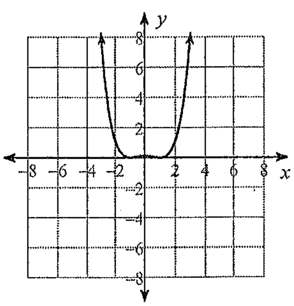
No vertical asymptotes exist.
No horizontal asymptotes exist.
Critical points at: $x = -\sqrt{2}, 0, \sqrt{2}$
Inflection points at: $x = -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}$

B)



x-intercepts at $x = -\sqrt{5}, 0, \sqrt{5}$ y-intercept at $y = 0$
No vertical asymptotes exist.
No horizontal asymptotes exist.
Critical points at: $x = -\frac{\sqrt{10}}{2}, 0, \frac{\sqrt{10}}{2}$
Inflection points at: $x = -\frac{\sqrt{30}}{6}, \frac{\sqrt{30}}{6}$

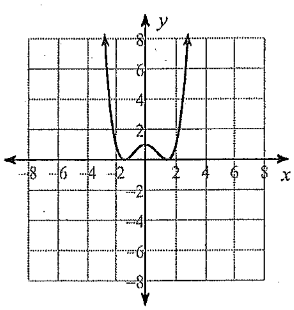
C)



x-intercepts at $x = -1, 1$ y-intercept at $y = \frac{1}{8}$

No vertical asymptotes exist.
No horizontal asymptotes exist.
Critical points at: $x = -1, 0, 1$
Inflection points at: $x = -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}$

D)



x-intercepts at $x = -\sqrt{2}, \sqrt{2}$ y-intercept at $y = 1$
No vertical asymptotes exist.
No horizontal asymptotes exist.
Critical points at: $x = -\sqrt{2}, 0, \sqrt{2}$
Inflection points at: $x = -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}$

