Name:________________________

Honors Calculus

Third Quarter Review

Directions: This serves as a review for your Third Quarter Test—please note that this test covers material from sections 2.1-2.8, 8.4, 3.1-3.4. I recommend that you attempt/answer all these questions as they are similar to the ones on your Third Quarter Test. In addition, please review your quizzes from this unit as well as homework assignments and notes completed in class. On the test, you will need a #2 pencil and you will be able to use a calculator.

Differentiate each function with respect to x.

1) $y=(-5x^3-2)\cdot 4x^4$

A) $\frac{dy}{dx}=-80x^6-32x^3$

B) $\frac{dy}{dx}=-20x^7-240x^5-8x^4$

C) $\frac{dy}{dx}=4x^4+16x^3$

D) $\frac{dy}{dx}=-140x^6-32x^3$

2) $y=\frac{3}{2x^5+2}$

A) $\frac{dy}{dx}=-\frac{15x^4}{2x^{10}+4x^5+2}$

B) $\frac{dy}{dx}=-\frac{15x^4}{x^5+1}$

C) $\frac{dy}{dx}=30x^4$

D) $\frac{dy}{dx}=-\frac{10x^4}{3}$

3) $f(x)=-4x+4x^{\frac{4}{5}}+\frac{1}{4}x^{\frac{3}{4}}$

A) $f'(x)=-4x+\frac{16x^{\frac{4}{5}}}{5}+\frac{3x^{\frac{3}{4}}}{16}$

B) $f'(x)=-4+\frac{4}{x^{\frac{1}{5}}}+\frac{1}{4x^{\frac{1}{4}}}$

C) $f'(x)=-4x+\frac{16x}{5}+\frac{3x}{16}$

D) $f'(x)=-4+\frac{16}{5x^{\frac{1}{5}}}+\frac{3}{16x^{\frac{1}{4}}}$

4) $y=(-3x^3+1)^{\frac{1}{3}}$

A) $\frac{dy}{dx}=\frac{1}{3}(-3x^3+1)^{-\frac{2}{3}}$
$=\frac{1}{3(-3x^3+1)^{\frac{2}{3}}}$

B) $\frac{dy}{dx}=-9x^2$

C) $\frac{dy}{dx}=(-3x^3+1)^{-\frac{2}{3}}\cdot -9x^2$
$=-\frac{9x^2}{(-3x^3+1)^{\frac{2}{3}}}$

D) $\frac{dy}{dx}=\frac{1}{3}(-3x^3+1)^{-\frac{2}{3}}\cdot -9x^2$
$=-\frac{3x^2}{(-3x^3+1)^{\frac{2}{3}}}$

5) $y = \csc 4x^3$

A) $\frac{dy}{dx} = -\csc 4x^3 \cot 4x^3 \cdot 12x^2$
$= -12x^2 \csc 4x^3 \cot 4x^3$

B) $\frac{dy}{dx} = -\csc 4x^3 \csc 4x^3 \cdot 12x^2$
$= -12x^2 \csc^2 4x^3$

C) $\frac{dy}{dx} = \csc 4x^3 \cot 4x^3 \cdot 12x^2$
$= 12x^2 \csc 4x^3 \cot 4x^3$

D) $\frac{dy}{dx} = -\csc 4x^3 \tan 4x^3 \cdot 12x^2$
$= -12x^2 \csc 4x^3 \tan 4x^3$

6) $f(x) = \csc 2x^3$

A) $f'(x) = -\csc 2x^3 \tan 2x^3 \cdot 6x^2$
$= -6x^2 \csc 2x^3 \tan 2x^3$

B) $f'(x) = -\csc 2x^3 \csc 2x^3 \cdot 6x^2$
$= -6x^2 \csc^2 2x^3$

C) $f'(x) = -\csc 2x^3 \cot 2x^3 \cdot 6x^2$
$= -6x^2 \csc 2x^3 \cot 2x^3$

D) $f'(x) = \csc 2x^3 \cot 2x^3 \cdot 6x^2$
$= 6x^2 \csc 2x^3 \cot 2x^3$

7) $f(x) = \sec 4x^4$

A) $f'(x) = \sec 4x^4 \cot 4x^4 \cdot 16x^3$
$= 16x^3 \sec 4x^4 \cot 4x^4$

B) $f'(x) = -\sec 4x^4 \tan 4x^4 \cdot 16x^3$
$= -16x^3 \sec 4x^4 \tan 4x^4$

C) $f'(x) = \sec 4x^4 \sec 4x^4 \cdot 16x^3$
$= 16x^3 \sec^2 4x^4$

D) $f'(x) = \sec 4x^4 \tan 4x^4 \cdot 16x^3$
$= 16x^3 \sec 4x^4 \tan 4x^4$

For each problem, find the derivative of the function at the given value.

8) $y = -\frac{3}{x^2 - 4}$ at $x = -1$

A) $\left.\frac{dy}{dx}\right|_{x=-1} = \frac{2}{3}$

B) $\left.\frac{dy}{dx}\right|_{x=-1} = -\frac{1}{6}$

C) $\left.\frac{dy}{dx}\right|_{x=-1} = -\frac{2}{3}$

D) $\left.\frac{dy}{dx}\right|_{x=-1} = \frac{18}{25}$

9) $y = \frac{2}{x + 3}$ at $x = 3$

A) $\left.\frac{dy}{dx}\right|_{x=3} = -\frac{2}{9}$

B) $\left.\frac{dy}{dx}\right|_{x=3} = -2$

C) $\left.\frac{dy}{dx}\right|_{x=3} = -\frac{2}{49}$

D) $\left.\frac{dy}{dx}\right|_{x=3} = -\frac{1}{18}$

For each problem, find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form.

10) $y = (x+2)^{\frac{1}{2}}$ at $(-1, 1)$

A) $y = \frac{1}{2}x + \frac{3}{2}$

B) $y = \frac{\sqrt{2}}{4}x + \sqrt{2}$

C) $y = \frac{1}{5}x + 3$

D) $y = \frac{1}{4}x + 1$

11) $y = \frac{9x}{x^2+9}$ at $\left(1, \frac{9}{10}\right)$

A) $y = \frac{18}{25}x + \frac{9}{50}$

B) $y = \frac{45}{169}x - \frac{144}{169}$

C) $y = -\frac{63}{625}x - \frac{1152}{625}$

D) $y = \frac{3}{2}$

12) $y = \frac{x^2}{2} + x + \frac{1}{2}$ at $\left(-2, \frac{1}{2}\right)$

A) $y = 5x - \frac{15}{2}$

B) $y = 0$

C) $y = 3x - \frac{3}{2}$

D) $y = -x - \frac{3}{2}$

For each problem, find the indicated derivative with respect to *x*.

13) $y = -2x^5 + 2x^3$ Find $\frac{d^2y}{dx^2}$

A) $\frac{d^2y}{dx^2} = -4x$

B) $\frac{d^2y}{dx^2} = -40x^3 + 12x$

C) $\frac{d^2y}{dx^2} = -50x^5 + 18x^3$

D) $\frac{d^2y}{dx^2} = -2x^3 + 2x$

14) $y = -x^3 + x$ Find $\frac{d^2y}{dx^2}$

A) $\frac{d^2y}{dx^2} = -9x^3 + x$

B) $\frac{d^2y}{dx^2} = -x + 2$

C) $\frac{d^2y}{dx^2} = -2x$

D) $\frac{d^2y}{dx^2} = -6x$

15) $y = 5x^5 + 2x^4 + 3x^3$ Find $\frac{d^3y}{dx^3}$

A) $\frac{d^3y}{dx^3} = 42x$

B) $\frac{d^3y}{dx^3} = 5x^2 + 2x + 3$

C) $\frac{d^3y}{dx^3} = 300x^2 + 48x + 18$

D) $\frac{d^3y}{dx^3} = 625x^5 + 128x^4 + 81x^3$

16) $y = 4x^2$ Find $\frac{d^3y}{dx^3}$

A) $\frac{d^3y}{dx^3} = -4$

B) $\frac{d^3y}{dx^3} = 0$

C) $\frac{d^3y}{dx^3} = 8x$

D) $\frac{d^3y}{dx^3} = 32x^2$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of *x* and *y*.

17) $y = 3x^3 + y^3$

A) $\frac{dy}{dx} = \frac{1 - 3y^2}{9x^2}$

B) $\frac{dy}{dx} = \frac{9x^2}{1 - 3y^2}$

C) $\frac{dy}{dx} = \frac{9x^2}{3x^3 + y^3}$

D) $\frac{dy}{dx} = \frac{y}{3x^3 + y^3}$

18) $4x - 2y^2 = 2$

A) $\frac{dy}{dx} = y$

B) $\frac{dy}{dx} = -2$

C) $\frac{dy}{dx} = \frac{1}{y}$

D) $\frac{dy}{dx} = 2x - y^2$

Solve each related rate problem.

19) A hypothetical square grows so that the length of its diagonals are increasing at a rate of 4 m/min. How fast is the area of the square increasing when the diagonals are 3 m each?

A) A = area of square x = length of diagonals t = time

Equation: $A = \frac{x^2}{2}$ Given rate: $\frac{dx}{dt} = 4$ Find: $\left.\frac{dA}{dt}\right|_{x=3}$

$$\left.\frac{dA}{dt}\right|_{x=3} = x \cdot \frac{dx}{dt} = 10 \text{ m}^2/\text{min}$$

B) A = area of square x = length of diagonals t = time

Equation: $A = \frac{x^2}{2}$ Given rate: $\frac{dx}{dt} = 4$ Find: $\left.\frac{dA}{dt}\right|_{x=3}$

$$\left.\frac{dA}{dt}\right|_{x=3} = x \cdot \frac{dx}{dt} = 14 \text{ m}^2/\text{min}$$

C) A = area of square x = length of diagonals t = time

Equation: $A = \frac{x^2}{2}$ Given rate: $\frac{dx}{dt} = 4$ Find: $\left.\frac{dA}{dt}\right|_{x=3}$

$$\left.\frac{dA}{dt}\right|_{x=3} = x \cdot \frac{dx}{dt} = 12 \text{ m}^2/\text{min}$$

D) A = area of square x = length of diagonals t = time

Equation: $A = \frac{x^2}{2}$ Given rate: $\frac{dx}{dt} = 4$ Find: $\left.\frac{dA}{dt}\right|_{x=3}$

$$\left.\frac{dA}{dt}\right|_{x=3} = x \cdot \frac{dx}{dt} = 7 \text{ m}^2/\text{min}$$

20) A spherical balloon is inflated at a rate of $\frac{32\pi}{3}$ cm³/sec. How fast is the radius of the balloon increasing when the radius is 4 cm?

A) V = volume of sphere r = radius t = time

Equation: $V = \frac{4}{3}\pi r^3$ Given rate: $\frac{dV}{dt} = \frac{32\pi}{3}$ Find: $\left.\frac{dr}{dt}\right|_{r=4}$

$$\left.\frac{dr}{dt}\right|_{r=4} = \frac{1}{4\pi r^2} \cdot \frac{dV}{dt} = \frac{1}{6} \text{ cm/sec}$$

B) V = volume of sphere r = radius t = time

Equation: $V = \frac{4}{3}\pi r^3$ Given rate: $\frac{dV}{dt} = \frac{32\pi}{3}$ Find: $\left.\frac{dr}{dt}\right|_{r=4}$

$$\left.\frac{dr}{dt}\right|_{r=4} = \frac{1}{4\pi r^2} \cdot \frac{dV}{dt} = \frac{2}{3} \text{ cm/sec}$$

C) V = volume of sphere r = radius t = time

Equation: $V = \frac{4}{3}\pi r^3$ Given rate: $\frac{dV}{dt} = \frac{32\pi}{3}$ Find: $\left.\frac{dr}{dt}\right|_{r=4}$

$$\left.\frac{dr}{dt}\right|_{r=4} = \frac{1}{4\pi r^2} \cdot \frac{dV}{dt} = \frac{1}{3} \text{ cm/sec}$$

D) V = volume of sphere r = radius t = time

Equation: $V = \frac{4}{3}\pi r^3$ Given rate: $\frac{dV}{dt} = \frac{32\pi}{3}$ Find: $\left.\frac{dr}{dt}\right|_{r=4}$

$$\left.\frac{dr}{dt}\right|_{r=4} = \frac{1}{4\pi r^2} \cdot \frac{dV}{dt} = \frac{5}{3} \text{ cm/sec}$$

For each problem, find all points of absolute minima and maxima on the given interval.

21) $y = \frac{2}{x^2 - 9}$; $[-7, -2]$

A) No absolute minima.
No absolute maxima.

B) Absolute minimum: $\left(0, -\frac{2}{9}\right)$
No absolute maxima.

C) Absolute minimum: $\left(-2, -\frac{2}{5}\right)$
Absolute maximum: $\left(0, -\frac{2}{9}\right)$

D) No absolute minima.
Absolute maximum: $\left(0, -\frac{2}{9}\right)$

22) $y = \frac{3}{x^2 - 16}$; $[-2, 3]$

A) Absolute minimum: $\left(3, -\frac{3}{7}\right)$

Absolute maximum: $\left(0, -\frac{3}{16}\right)$

B) Absolute minimum: $\left(0, -\frac{3}{16}\right)$

Absolute maximum: $\left(-2, -\frac{1}{4}\right)$

C) Absolute minimum: $\left(0, -\frac{3}{16}\right)$

Absolute maximum: $\left(3, -\frac{3}{7}\right)$

D) No absolute minima.
No absolute maxima.

For each problem, find all points of relative minima and maxima.

23) $y = x^4 - x^2 - 2$

A) No relative minima.
Relative maxima: $\left(-\frac{\sqrt{2}}{6}, -\frac{665}{324}\right)$, $\left(\frac{1}{3}, -\frac{170}{81}\right)$, $\left(\frac{\sqrt{2}}{6}, -\frac{665}{324}\right)$

B) No relative minima.
No relative maxima.

C) Relative minimum: $(4, 238)$
Relative maxima: $(-2\sqrt{2}, 54)$, $(2\sqrt{2}, 54)$

D) Relative minima: $\left(-\frac{\sqrt{2}}{2}, -\frac{9}{4}\right)$, $\left(\frac{\sqrt{2}}{2}, -\frac{9}{4}\right)$
Relative maximum: $(0, -2)$

24) $y = -x^4 + x^2$

A) No relative minima.
No relative maxima.

B) Relative minima: $\left(-\frac{\sqrt{2}}{6}, \frac{17}{324}\right)$, $\left(\frac{1}{3}, \frac{8}{81}\right)$
Relative maximum: $\left(\frac{\sqrt{2}}{6}, \frac{17}{324}\right)$

C) Relative minima: $(-2\sqrt{2}, -56)$, $(4, -240)$, $(2\sqrt{2}, -56)$
No relative maxima.

D) Relative minimum: $(0, 0)$
Relative maxima: $\left(-\frac{\sqrt{2}}{2}, \frac{1}{4}\right)$, $\left(\frac{\sqrt{2}}{2}, \frac{1}{4}\right)$

For each problem, find the open intervals where the function is increasing and decreasing.

25) $y = -2x^2 - 12x - 19$

A) Increasing: $(-1, \infty)$ Decreasing: $(-\infty, -1)$
B) Increasing: $(-3, \infty)$ Decreasing: $(-\infty, -3)$
C) Increasing: $(-\infty, -3)$ Decreasing: $(-3, \infty)$
D) Increasing: $(-\infty, -12)$ Decreasing: $(-12, \infty)$

26) $y = -x^2 - 1$

A) Increasing: $(-\infty, 0)$ Decreasing: $(0, \infty)$
B) Increasing: $\left(\frac{1}{3}, \infty\right)$ Decreasing: $\left(-\infty, \frac{1}{3}\right)$
C) Increasing: $(-\infty, 4)$ Decreasing: $(4, \infty)$
D) Increasing: $(0, \infty)$ Decreasing: $(-\infty, 0)$

For each problem, find the open intervals where the function is concave up and concave down.

27) $y = -x^3 + x^2 + 2$

A) Concave up: $\left(\frac{1}{3}, \infty\right)$ Concave down: $\left(-\infty, \frac{1}{3}\right)$
B) Concave up: $\left(\frac{1}{9}, \infty\right)$ Concave down: $\left(-\infty, \frac{1}{9}\right)$
C) Concave up: $\left(-\infty, \frac{4}{3}\right)$ Concave down: $\left(\frac{4}{3}, \infty\right)$
D) Concave up: $\left(-\infty, \frac{1}{3}\right)$ Concave down: $\left(\frac{1}{3}, \infty\right)$

28) $y = x^4 - 4x^2 - 1$

A) Concave up: $\left(-\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}\right)$ Concave down: $\left(-\infty, -\frac{\sqrt{6}}{3}\right), \left(\frac{\sqrt{6}}{3}, \infty\right)$
B) Concave up: $\left(-\frac{\sqrt{6}}{9}, \frac{\sqrt{6}}{9}\right)$ Concave down: $\left(-\infty, -\frac{\sqrt{6}}{9}\right), \left(\frac{\sqrt{6}}{9}, \infty\right)$
C) Concave up: $\left(-\infty, -\frac{\sqrt{6}}{3}\right), \left(\frac{\sqrt{6}}{3}, \infty\right)$ Concave down: $\left(-\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}\right)$
D) Concave up: $\left(-\infty, -\frac{4\sqrt{6}}{3}\right), \left(\frac{4\sqrt{6}}{3}, \infty\right)$ Concave down: $\left(-\frac{4\sqrt{6}}{3}, \frac{4\sqrt{6}}{3}\right)$

Solve each optimization problem.

29) A company has started selling a new type of smartphone at the price of $\$150 - 0.1x$ where x is the number of smartphones manufactured per day. The parts for each smartphone cost \$80 and the labor and overhead for running the plant cost \$5000 per day. How many smartphones should the company manufacture and sell per day to maximize profit?

A) 400 B) 650 C) 350 D) 750

30) A supermarket employee wants to construct an open-top box from a 14 by 30 in piece of cardboard. To do this, the employee plans to cut out squares of equal size from the four corners so the four sides can be bent upwards. What size should the squares be in order to create a box with the largest possible volume?

A) 6 in B) 4 in C) 3 in D) 7 in

